

ENGINEERING MATHEMATICS II

IMPORTANT UNIVERSITY PROBLEMS FOR
MA 8251 MATHEMATICS –I

UNIT -1
MATRICES
PART –A

1. Find the characteristic Equation of $\begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$.

Solution

Let $A = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix}$. The characteristic Equation is $|A - \lambda I| = 0$

$$\left[\begin{bmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right] = 0 \Rightarrow \begin{vmatrix} 2-\lambda & -3 & 1 \\ 3 & 1-\lambda & 3 \\ -5 & 2 & -4-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)[(1-\lambda)(-4-\lambda)-6] + 3[3(-4-\lambda)+15] + 1[6+5(1-\lambda)] = 0$$

$$(2-\lambda)[-10-\lambda+4\lambda+\lambda^2] + 3[3-3\lambda] + [6+5-5\lambda] = 0$$

$$2\lambda^2 + 6\lambda - 20 - \lambda^3 - 3\lambda^2 + 4\lambda - 9\lambda + 9 + 11 - 5\lambda = 0 \Rightarrow \lambda^3 + \lambda^2 - 2\lambda = 0$$

2. Given $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$. Find the eigen values of A^2 .

Solution

Given matrix A is a lower triangular matrix. We know that “The eigen value of a triangular matrix is the diagonal elements”. From this, the Eigen values of A^2 are $(-1)^2, (-3)^2, (2)^2$. The Eigen values of $A^2 = 1, 9, 4$.

3. State Cayley – Hamilton theorem and verify it for the matrix $A = \begin{bmatrix} 3 & -1 \\ -1 & 5 \end{bmatrix}$

Solution Cayley – Hamilton Theorem: Every square matrix A satisfies its own characteristic equation.

Verification:

$A = \begin{bmatrix} 3 & -1 \\ -1 & 5 \end{bmatrix}$ The characteristic Equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & -1 \\ -1 & 5-\lambda \end{vmatrix} = 0 \Rightarrow (3-\lambda)(5-\lambda) - 1 = 0 \Rightarrow 15 - 3\lambda - 5\lambda + \lambda^2 - 1 = 0 \Rightarrow \lambda^2 - 8\lambda + 14 = 0$$

To prove $A^2 - 8A + 14I = 0$

$$A^2 = \begin{bmatrix} 3 & -1 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 10 & -8 \\ -8 & 26 \end{bmatrix}$$

$$A^2 - 8A + 14I = \begin{bmatrix} 10 & -8 \\ -8 & 26 \end{bmatrix} - 8 \begin{bmatrix} 3 & -1 \\ -1 & 5 \end{bmatrix} + 14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & -8 \\ -8 & 26 \end{bmatrix} - \begin{bmatrix} 24 & -8 \\ -8 & 40 \end{bmatrix} + \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$\Rightarrow A^2 - 8A + 14I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ Hence the given matrix satisfies Cayley – Hamilton Theorem.}$$

4. Using Cayley – Hamilton Theorem find A^{-1} if $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$.

Solution $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$, The characteristic Equation is $|A - \lambda I| = 0$

$$|A - \lambda I| = \begin{vmatrix} 2 - \lambda & -1 \\ 4 & 3 - \lambda \end{vmatrix} = [(2 - \lambda)(3 - \lambda) + 4] = 6 - 2\lambda - 3\lambda + \lambda^2 + 4 = \lambda^2 - 5\lambda + 10$$

By Cayley – Hamilton Theorem $A^2 - 5A + 10I = 0$ (1)

To find A^{-1} multiply equation (1) by A^{-1} .

$$A - 5I + 10A^{-1} = 0 \Rightarrow \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 10A^{-1} = 0 \Rightarrow \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} + 10A^{-1} = 0 \Rightarrow \begin{bmatrix} -3 & -1 \\ 4 & -2 \end{bmatrix} + 10A^{-1} = 0$$

$$\Rightarrow 10A^{-1} = - \begin{bmatrix} -3 & -1 \\ 4 & -2 \end{bmatrix} \Rightarrow A^{-1} = -\frac{1}{10} \begin{bmatrix} -3 & -1 \\ 4 & -2 \end{bmatrix}$$

5. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ find A^3 using Cayley – Hamilton Theorem.

Solution The characteristic equation is $|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1 - \lambda & 2 \\ 3 & 4 - \lambda \end{vmatrix} = 0 \Rightarrow$

$$\Rightarrow \lambda^2 - 5\lambda - 2 = 0$$

Using Cayley – Hamilton theorem we get, $A^2 - 5A - 2I = 0$

$$A^2 = A \times A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} \Rightarrow A^2 = 5A + 2I$$

$$\text{Multiply both sides by } A \text{ we get, } A^3 = 5A^2 + 2I = 5 \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} + 2 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow A^3 = \begin{bmatrix} 37 & 54 \\ 81 & 118 \end{bmatrix}$$

6. Prove that the similar matrices have same Eigen values.

Proof: Let A, B be two similar matrices. Then there exists a non – singular matrix P such that $B = P^{-1}AP$

$$B - \lambda I = P^{-1}AP - \lambda I = P^{-1}AP - P^{-1}\lambda P = P^{-1}(A - \lambda I)P$$

$$|B - \lambda I| = |P^{-1}| |A - \lambda I| |P| = |A - \lambda I| |P^{-1}P| = |A - \lambda I| |I| \Rightarrow |B - \lambda I| = |A - \lambda I|$$

Therefore A and B have the same characteristic functions and hence the same characteristic roots. Therefore they have the same eigen values.

7. Check whether the matrix P is orthogonal $P = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$

Solution Condition for orthogonal matrix is $PP^T = P^T P = I$

$$P = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \quad P^T = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \quad PP^T = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$PP^T = \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & -\cos \theta \sin \theta + \cos \theta \sin \theta \\ -\sin \theta \cos \theta + \cos \theta \sin \theta & \sin^2 \theta + \cos^2 \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Similarly, $P^T P = I$. Therefore the given matrix is orthogonal.

8. State Fundamental theorem of quadratic form.

Solution Any quadratic form may be reduced to canonical form by means of a Non- singular transformation

9. Check whether the matrix B is orthogonal? Justify =
$$\begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution Condition for orthogonal matrix is $BB^T = B^T B = I$

$$B = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow B^T = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$BB^T = \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & -\cos \theta \sin \theta + \cos \theta \sin \theta & 0 \\ -\cos \theta \sin \theta + \cos \theta \sin \theta & \cos^2 \theta + \sin^2 \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Similarly $B^T B = I$. Therefore the given matrix is orthogonal

10. Write the matrix of the quadratic form $2x_1^2 - 2x_2^2 + 4x_3^2 + 2x_1x_2 - 6x_1x_3 + 6x_2x_3$.

Solution

$$Q = \begin{bmatrix} \text{coeffi}(x_1^2) & \frac{1}{2}\text{coeffi}(x_1x_2) & \frac{1}{2}\text{coeffi}(x_1x_3) \\ \frac{1}{2}\text{coeffi}(x_2x_1) & \text{coeffi}(x_2^2) & \frac{1}{2}\text{coeffi}(x_2x_3) \\ \frac{1}{2}\text{coeffi}(x_3x_1) & \frac{1}{2}\text{coeffi}(x_3x_2) & \text{coeffi}(x_3^2) \end{bmatrix} \Rightarrow Q = \begin{bmatrix} 2 & 1 & -3 \\ 1 & -2 & 3 \\ -3 & 3 & 4 \end{bmatrix}$$

11. Write the quadratic form corresponding to the following symmetric matrix $\begin{bmatrix} 0 & -1 & 2 \\ -1 & 1 & 4 \\ 2 & 4 & 3 \end{bmatrix}$

Solution Let $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 2 \\ -1 & 1 & 4 \\ 2 & 4 & 3 \end{bmatrix}$

The general quadratic form is $a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2(a_{12})x_1x_2 + 2(a_{23})x_2x_3 + 2(a_{13})x_1x_3$

$$= 0.x_1^2 + 1.x_2^2 + 3.x_3^2 + 2(-1)x_1x_2 + 2(4)x_2x_3 + 2(2)x_1x_3$$

$$= x_2^2 + 3x_3^2 - 2x_1x_2 + 8x_2x_3 + 4x_1x_3$$

12. Determine the nature of the following quadratic form

$$f(x, y, z) = x^2 + 2y^2 + 3z^2 + 2xy + 2yz - 2xz$$

Solution The matrix of the Quadratic form is

$$Q = \begin{bmatrix} \text{coeffi}(x^2) & \frac{1}{2}\text{coeffi}(xy) & \frac{1}{2}\text{coeffi}(xz) \\ \frac{1}{2}\text{coeffi}(yx) & \text{coeffi}(y^2) & \frac{1}{2}\text{coeffi}(yz) \\ \frac{1}{2}\text{coeffi}(zx) & \frac{1}{2}\text{coeffi}(zy) & \text{coeffi}(z^2) \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$D_1 = |a_{11}| = |1| = 1 = +ve \quad D_2 = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1 = +ve \quad D_3 = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{vmatrix} = -2 = -ve$$

Therefore the quadratic form is indefinite.

13. If the sum of two eigenvalues and trace of a 3x3 matrix A are equal, find the value of $|A|$.

Solution Let $\lambda_1, \lambda_2, \lambda_3$ be the eigenvalues of the given 3x3 matrix A.

We know that Sum of the eigenvalues = trace of A

i.e. $\lambda_1 + \lambda_2 + \lambda_3 = \text{trace of A} \dots\dots\dots(1)$

Given, trace of A = sum of two eigenvalues = $\lambda_1 + \lambda_2$

$$\lambda_1 + \lambda_2 + \lambda_3 = \lambda_1 + \lambda_2 \Rightarrow \lambda_3 = 0$$

We know that $|A| = \text{product of the eigenvalues} = \lambda_1 \times \lambda_2 \times 0 = 0$

14. Can $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ be diagonalized? Why?

Solution The given matrix A is a real symmetric and non – singular matrix. Hence A is diagonalized.

15. If the eigenvalues of A are 1, 2, 3 then what are the eigenvalues of Adj A.

Solution Eigen values of A are 1, 2, 3

Eigen values of A^{-1} are $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}$ We know that $A^{-1} = \frac{adj A}{|A|} \Rightarrow adj A = |A|A^{-1}$

$|A|$ = product of eigenvalues. = $1 \times 2 \times 3 = 6 \Rightarrow |A| = 6$

Eigen values of adj A = $6(1), 6(\frac{1}{2}), 6(\frac{1}{3}) \Rightarrow$ Eigen values of adj A are 6, 3, 2.

16. Two of the eigenvalues of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ are 3 and 6. Find the eigenvalues of A^{-1} .

Solution Sum eigenvalues = sum of the main diagonal elements.

$$3 + 6 + \lambda_3 = 3 + 5 + 3 \Rightarrow 9 + \lambda_3 = 11 \Rightarrow \lambda_3 = 2$$

Therefore the eigenvalues of A are 3, 6, 2.

Therefore the eigenvalues of A^{-1} are $\frac{1}{3}, \frac{1}{6}, \frac{1}{2}$.

17. Prove that $A^3 - 20A + 8I = 0$ where $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

Solution $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ The characteristic equation of A is $|A - \lambda I| = 0$

$$\text{i.e., } \begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 3-\lambda & -3 \\ -2 & -4 & -4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)[(3-\lambda)(-4-\lambda)-12] - 1[1(4-\lambda)-6] + 3[-4+2(3-\lambda)] = 0$$

$$\Rightarrow -\lambda^3 + 20\lambda - 8 = 0 \Rightarrow \lambda^3 - 20\lambda + 8 = 0$$

By Cayley – Hamilton Theorem, $A^3 - 20A + 8I = 0$.

18. Find the eigenvector corresponding to the eigenvalue 1 of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

Solution Given eigenvalue is 1. To find the eigenvector solve $(A - \lambda I)X = 0$.

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \text{ Put } \lambda = 1 \text{ we get } \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$x_1 + 2x_2 + x_3 = 0 \dots\dots(1) \quad x_1 + 2x_2 + x_3 = 0 \dots\dots(2) \quad x_1 + 2x_2 + x_3 = 0 \dots\dots(3)$$

All the three equations are same $x_1 + 2x_2 + x_3 = 0$

$$\text{Put } x_1 = 0 \text{ we get } 2x_2 + x_3 = 0 \Rightarrow 2x_2 = -x_3$$

$$\frac{x_2}{-1} = \frac{x_3}{2}$$

Hence the eigenvectors corresponding to 1 is $X_1 = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$.

19. Find the product and sum of the eigenvalues of the matrix $A = \begin{pmatrix} 2 & -3 \\ 4 & -2 \end{pmatrix}$

Solution Sum of the eigenvalues = sum of the main diagonal elements = $2 - 2 = 0$

$$\text{Product of the eigenvalues} = |A| = \begin{vmatrix} 2 & -3 \\ 4 & -2 \end{vmatrix} = -4 + 12 = 8.$$

20. Determine the matrix A for which the eigenvalues and eigenvectors are 2, 3, 5 and

$(1, -1, 0)^T, (1, 0, 0)^T, (2, 2, 1)^T$ respectively.

Solution Forming the model matrix $B = \begin{pmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

We get $B^{-1}AB = D$ Where $D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ is the diagonal matrix

Pre multiplying by B and post multiplying by B^{-1} , we obtain $B B^{-1} A B B^{-1} = B D B^{-1} \Rightarrow A = B D B^{-1}$

Now $B^{-1} = \frac{\text{adj}B}{|B|}$

$$\text{adj}A = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{pmatrix}^T = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 1 & -5 \\ 0 & 0 & 1 \end{pmatrix}$$

$$|B| = \begin{vmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{vmatrix} = 1 [0-0] - 1 [-1+0] + 3 [0+0] = 1$$

$$B^{-1} = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 1 & -5 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A = B D B^{-1} = \begin{pmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} 0 & -1 & 2 \\ 1 & 1 & -5 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -2 & 4 \\ 3 & 3 & -15 \\ 0 & 0 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$$

21. If A is an orthogonal matrix show that A^{-1} is also orthogonal .

Solution For an orthogonal matrix, transpose will be the inverse.

Therefore $A^T = A^{-1}$ (1)

Let $A^T = A^{-1} = B$ (2)

Then $B^T = (A^{-1})^T = (A^T)^{-1} = B^{-1}$ [From (2)]

$B^T = B^{-1}$ matrix B is orthogonal. i.e. A^{-1} is also orthogonal.

22. The Product of two eigenvalues of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigenvalue of A.

Solution Let the eigen values of the matrix A be $\lambda_1, \lambda_2, \lambda_3$.

Given $\lambda_1 \lambda_2 = 16$

$$\text{We know that } \lambda_1 \lambda_2 \lambda_3 = |A| \Rightarrow \lambda_1 \lambda_2 \lambda_3 = \begin{vmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix} = 32 \Rightarrow$$

$$\lambda_1 \lambda_2 \lambda_3 = 32 \Rightarrow 16 \lambda_3 = 32 \Rightarrow \lambda_3 = 2$$

23. For a given matrix A of order 3, $|A|=32$ and two of its eigen values are 8 and 2. Find the sum of the eigen values.

Solution Given matrix A of order 3.

Let the eigen values are $\lambda_1, \lambda_2, \lambda_3$. W.K.T $|A| = \text{Product of eigen values} = \lambda_1 \lambda_2 \lambda_3$

Given: $|A|=32$ and $\lambda_1 = 8, \lambda_2 = 2$

$$\lambda_1 \lambda_2 \lambda_3 = 32 \Rightarrow (8)(2)\lambda_3 = 32 \Rightarrow \lambda_3 = 2 \quad \text{Sum of the eigen values} = \lambda_1 + \lambda_2 + \lambda_3 = 8 + 2 + 2 = 12$$

24. Use- Cayley Hamilton theorem to find $(A^4 - 4A^3 - 5A^2 + A + 2I)$ when $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

Solution Given $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

The characteristic equation of A is $|A - \lambda I| = 0$ That is $\lambda^2 - s_1 + s_2 = 0$ where

$s_1 =$ sum of the main diagonal elements $= 1+3=4$ and $s_2 = |A| = \begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} = 3-8 = -5$

Therefore the characteristic equation of A is $\lambda^2 - 4\lambda - 5 = 0$

By Cayley Hamilton theorem we get $A^2 - 4A - 5I = 0$

$$A^4 - 4A^3 - 5A^2 + A + 2I = A^2(A^2 - 4A - 5I) + A + 2I = A^2(0) + A + 2I = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$$

25. If 1 and 2 are the eigenvalues of a 2×2 matrix A. What are the eigenvalues of A^2 and A^{-1} .

Solution Given 1 and 2 are the eigenvalues of a 2×2 matrix A.

By property, 1^2 and $2^2=4$ are the eigenvalues of A^2 and 1 and $1/2$ are the eigenvalues of A^{-1} .

26. If the sum of the eigen values of the matrix of the quadratic form equal to zero, then what will be the nature of the quadratic form?

Solution Given $\lambda_1 + \lambda_2 + \lambda_3 = 0$

Case (i) All +ve is not possible, Case (ii) All -ve is not possible, Case (iii) both positive and negative is possible. Therefore Nature of the Quadratic form is indefinite.

27. A is a singular matrix of order three, 2 and 3 are the eigen values. Find its third eigen value

Solution Given $\lambda_1 = 2, \lambda_2 = 3, \lambda_3 = \text{_____}$

Given A is singular $\Rightarrow |A|=0$

$|A| =$ product of the eigen values $= 0 \Rightarrow (2)(3)(0) = 0 \therefore \lambda_3 = 0$

28. The sum of the eigen values of the matrix $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

Solution Sum of the eigen values = Sum of the main diagonal elements $= (2)+(1)+(-1)=2$

Product of the eigen values $= |A| = \begin{vmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -2 \end{vmatrix} = -8$

29. Using Cayley - Hamilton theorem, is it possible to find the inverse of all square matrices? Justify your answer.

Solution Not possible to find the inverse of all square matrices by using Cayley- Hamilton theorem.

Let A be any square matrix of order n.

The characteristic equation of A is by Cayley - Hamilton theorem we get

$$A^n + C_{n-1}A^{n-1} + C_{n-2}A^{n-2} + \dots + C_1A + C_0 = 0$$

A^{-1} exists only if $C_0 = |A| \neq 0$

30. Sum of the squares of the eigen values of $\begin{bmatrix} 1 & 7 & 5 \\ 0 & 2 & 9 \\ 0 & 0 & 5 \end{bmatrix}$ is

Solution Given is an upper triangular matrix.

Hence the eigen values are 1, 2, 5.

Sum of squares of eigen values $= 1^2 + 2^2 + 5^2 = 30$

31. Find the sum and product of eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & -3 \end{bmatrix}$

Solution: Sum of the matrix = Sum of the diagonal elements $= 1+0-3 = -2$

Product of the eigen values= determinant value of the matrix =1(0+3)-2(-3+6)-2(-1+0)=3-6+2= -1.

32. What is the diagonalisation of a symmetric matrix ?

Solution If a square matrix of order n has n linearly independent eigen values, then a

Matrix P can be found such that $P^{-1}AP$ is a diagonal matrix.

33. If $A = \begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}$, express A^3 in terms of A and I Cayley-Hamilton theorem.

Solution The characteristic equation of the given matrix is

$$|A - \lambda I| = 0 \Rightarrow \left| \begin{pmatrix} 1 & 0 \\ 4 & 5 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 0 \\ 4 & 5-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(5-\lambda) - 0 = 0 \Rightarrow 5 - \lambda - 5\lambda + \lambda^2 = 0 \Rightarrow \lambda^2 - 6\lambda + 5 = 0$$

By Cayley-Hamilton theorem. [Every square matrix A satisfies its own char. equation]

$$A^2 - 6A + 5I = 0 \Rightarrow A^2 = 6A - 5I$$

$$\text{Multiply } A \text{ on both sides} \Rightarrow A^3 - 6A^2 + 5I = 0 = 6[6A - 5I] - 5A = 36A - 30I - 5A = 31A - 30I$$

34. Find the nature of the quadratic form $2x^2 + 2xy + 3y^2$

Solution The matrix of the quadratic form is

$$Q = \begin{bmatrix} \text{coeff}x^2 & \frac{1}{2}\text{coeff}.xy \\ \frac{1}{2}\text{coeff}.yx & \text{coeff}y^2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \Rightarrow D_1 = |2| = 2(+ve) \text{ and } D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 6 - 1 = 5(+ve)$$

Therefore the QF is positive definite.

35. State the nature of the Q.F $2xy + 2yz + 2zx$.

Solution

$$Q = \begin{bmatrix} \text{Coeff}x^2 & \frac{1}{2}\text{Coeff}x.y & \frac{1}{2}\text{Coeff}x.z \\ \frac{1}{2}\text{Coeff}z.x & \text{Coeff}y^2 & \frac{1}{2}\text{Coeff}y.z \\ \frac{1}{2}\text{Coeff}z.x & \frac{1}{2}\text{Coeff}z.y & \text{Coeff}z^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

36. Write the quadratic form corresponding to the following symmetric matrix

$$\begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & 2 & 3 \end{bmatrix}$$

Solution Let $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 2 & -2 \\ -2 & 2 & 3 \end{bmatrix}$

The general quadratic form is $a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2(a_{12})x_1x_2 + 2(a_{23})x_2x_3 + 2(a_{13})x_1x_3$
 $= 2x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 - 4x_2x_3 - 4x_1x_3$

37. Find the eigen values of the matrix A^{-1} if $A = \begin{bmatrix} 1 & 5 \\ 0 & 4 \end{bmatrix}$

Solution Given $A = \begin{bmatrix} 1 & 5 \\ 0 & 4 \end{bmatrix}$ Given is an upper triangular matrix. Then the property of the eigen

values of A are 1, 4

Therefore the eigen values of A^{-1} are $1/1, 1/4 \Rightarrow 1, 1/4$

38. Find the eigen values of A^{-1} if the matrix A is $\begin{bmatrix} 2 & 5 & -1 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

Solution Clearly given A is a upper triangular matrix. Hence the eigen values are 2,3,4

By the property the eigen values of A^{-1} are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.

39. If 2 is an eigen values of $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ find the other two values

Solution Let $\lambda_1, \lambda_2, \lambda_3$ are the eigen values pf A.

Given $\lambda_1 = 2$, to find : λ_2 and λ_3 We know that $\lambda_1 + \lambda_2 + \lambda_3 = (2)+(1)+(-1)=2 \Rightarrow$

$$\lambda_2 + \lambda_3 = 0$$

$$\lambda_1, \lambda_2, \lambda_3 = |A| = \begin{vmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -2 \end{vmatrix} = -8$$

$$\lambda_1, \lambda_2 = -4 \Rightarrow \lambda_2(-\lambda_2) = -4 \Rightarrow \lambda_2^2 = -4 \Rightarrow \lambda_2 = 2 \Rightarrow 2 + \lambda_3 = -0 \Rightarrow \lambda_3 = -2$$

The other two eigen values are 2,-2.

40. Find the sum and product of eigen values of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

Solution: Clearly given is an upper triangular matrix. Hence the eigen values are 3,2,5.

Sum of the eigen values = 3,2,5 and Product of the eigen values = (3)(2)(5) = 30

41. Show that $ax^2_1 - 2bx_1x_2 + cx^2_2$ is positive definite if and only if $a > 0$ and $ac - b^2$

Solution The matrix of the quadratic form is

$$Q = \begin{bmatrix} \text{coeff}x_1^2 & \frac{1}{2}\text{coeff}x_1x_2 \\ \frac{1}{2}\text{coeff}x_2x_1 & \text{coeff}x_2^2 \end{bmatrix} = \begin{bmatrix} a & -b \\ -b & c \end{bmatrix}$$

$$D_1 = |a| = a > 0 \text{ and } D_2 = \begin{vmatrix} a & -b \\ -b & c \end{vmatrix} = ac - b^2$$

Q.F is positive definite if $a > 0$ and $ac - b^2 > 0$ That is $a > 0$ and $ac > b^2$

42. Find the eigen value of the matrix of $A = \begin{bmatrix} 1 & 1 \\ 3 & -3 \end{bmatrix}$

Solution: Let $A = \begin{bmatrix} 1 & 1 \\ 3 & -3 \end{bmatrix}$

$$\text{The characteristic equation of A is } |A - \lambda I| = 0 \Rightarrow \lambda^2 - S_1\lambda + S_2 = 0$$

$$S_1 = \text{Sum of the main diagonal elements} = (1) + (-1)$$

$$S_2 = |A| = \begin{vmatrix} 1 & 1 \\ 3 & -1 \end{vmatrix} = -1 - 3 = -4$$

$$\text{Therefore the characteristic equation is } \lambda^2 - 0\lambda + (-4) = 0 \Rightarrow \lambda^2 - 4 = 0 \Rightarrow \lambda^2 = 4 \Rightarrow \lambda = \pm 2$$

43. The characteristic root of a triangular matrix are just the diagonal elements of the matrix .(or)

The eigen values of a triangular matrix are just the diagonal elements of the matrix .

Proof: Let us consider the triangular matrix. $A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$\text{Characteristic equation of A is } |A - \lambda I| = 0$$

$$A = \begin{bmatrix} a_{11} - \lambda & 0 & 0 \\ a_{21} & a_{22} - \lambda & 0 \\ a_{31} & a_{32} & a_{33} - \lambda \end{bmatrix} = 0$$

On expansion it gives $(a_{11} - \lambda)(a_{22} - \lambda)(a_{33} - \lambda) = 0$

$\lambda = a_{11}, a_{22}, a_{33}$, which are diagonal elements of matrix A.

44. If λ is an Eigen value of a matrix A, then $1/\lambda$, ($\lambda \neq 0$) is the eigen value of A^{-1} (OR) If λ is an Eigen value of a matrix A, what can you say about the Eigen of matrix A^{-1} . Prove your statement.

Proof: If X be the eigenvector corresponding to λ , Then $AX = \lambda X$ (1)

Premultiplying both sides by A^{-1} , we

$$\text{get } A^{-1}AX = A^{-1}\lambda X \Rightarrow IX = \lambda A^{-1}X \Rightarrow \frac{1}{\lambda}X = A^{-1}X \Rightarrow A^{-1}X = \frac{1}{\lambda}X$$

This being of the same form as (i), shows that $1/\lambda$ is an Eigen value of the inverse matrix A^{-1} .

45. Find the eigen values of the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 4 & 4 \end{bmatrix}$.

Solution: Given $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 4 & 4 \end{bmatrix}$

Clearly given matrix A is a lower triangular matrix, hence by the property of the eigen values of A are 2,3,4.

46. Define orthogonal matrices.

Solution A square matrix A is said to be orthogonal if $AA^T = A^T A = I$:: $AA^{-1} = A^{-1}A = I \Rightarrow A$ is orthogonal if $A^T = A^{-1}$

PART - B

1. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$.

2. Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$. Hence find its inverse.

[Nov/Dec 2016]

3. Diagonalize $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by an orthogonal transformation. [Nov/Dec 2016]

4. Reduce the quadratic form $2x^2 + 5y^2 + 3z^2 + 4xy$ to canonical form by an orthogonal transformation. Determine its nature.

5. Reduce the quadratic form $q = 3x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3$ to its normal form by an orthogonal reduction.

6. Reduce the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$, to the canonical form through an orthogonal transformation and hence show that it is positive semi-definite.

7. Verify Cayley-Hamilton theorem for the matrix and find A^{-1} and A^4 . $A = \begin{pmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{pmatrix}$

8. Reduce the matrix $\begin{bmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{bmatrix}$ to diagonal form.

9. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$

10. Reduce the quadratic form $x_1^2 + 2x_2x_3$ into a canonical form by means of an orthogonal transformation. Find a set of non-zero values x_1, x_2 & x_3 for which the above quadratic form is zero.

11. (i) Find the characteristic equation of the matrix A. Given $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence find A^{-1} and A^4

(ii) find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

12. (i) Use Cayley-Hamilton theorem to find the value of the matrix given by

$$(A^8 - 5A^7 + 7A^6 - 3A^5 + 8A^4 + 5A^3 + 8A^2 - 2A + I), \text{ if the matrix } A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

(ii) find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

13. Reduce the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ to the canonical form through an orthogonal transformation and hence show that it is positive semi-definite. Also give a non-zero set of values (x_1, x_2, x_3) which makes this quadratic form zero.

14. Reduce the quadratic form to $x^2 + 3y^2 + 3z^2 - 2yz$ canonical form by an orthogonal

transformation.

15. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$

16. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

17. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

18. The eigen vectors of a 3×3 real symmetric matrix A corresponding to the eigen values 2,3,6 are $[1,0,-1]^T$, $[1,1,1]^T$ and $[-1,2,-1]^T$ respectively find the matrix A.

19. Diagonalize $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ by an orthogonal transformation.

20. Reduce the quadratic form $Q = 6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$ to canonical form by an orthogonal transformation. [Nov/Dec 2016]

21. Reduce the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1$ to the canonical form through an orthogonal transformation.

22. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form through an orthogonal transformation.

23. Reduce the quadratic form $x^2 + y^2 + z^2 - 2yz - 2zx - 2xy$ to the canonical form through an orthogonal transformation. [May/June 2012]

24. Reduce the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ to diagonal form.

25. Verify Cayley-Hamilton theorem for the matrix and find A^{-1} if $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ [May/June 2016]

UNIT-II
VECTOR CALCULUS
PART-A

1. Is the vector $x\vec{i} + 2y\vec{j} + 3z\vec{k}$, Irrotational?

(AU-2009)

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & 2y & 3z \end{vmatrix} = \vec{i}(0-0) - \vec{j}(0) + \vec{k}(0-0) = 0$$

$\therefore \vec{F}$ is irrotational.

2. Find the $\text{divcurl } \vec{F} = x^2y\vec{i} + xz\vec{j} + 2yz\vec{k}$

(AU-2010)

$$\begin{aligned} \text{Curl } \vec{F} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & xz & 2yz \end{vmatrix} = \vec{i} \left[\frac{\partial}{\partial y}(2yz) - \frac{\partial}{\partial z}(xz) \right] - \vec{j} \left[\frac{\partial}{\partial x}(2yz) - \frac{\partial}{\partial z}(x^2y) \right] + \vec{k} \left[\frac{\partial}{\partial x}(xz) - \frac{\partial}{\partial y}(x^2y) \right] \\ &= \vec{i}(2z-x) - \vec{j}(0) + \vec{k}(z-x^2) \end{aligned}$$

$$\begin{aligned} \text{Divcurl } \vec{F} &= \nabla \cdot \text{curl } \vec{F} \\ &= \nabla \cdot (\vec{i}(2z-x) - \vec{j}(0) + \vec{k}(z-x^2)) \\ &= \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot (\vec{i}(2z-x) - \vec{j}(0) + \vec{k}(z-x^2)) \\ &= \left(\frac{\partial}{\partial x}(2z-x) + \frac{\partial}{\partial z}(z-x^2) \right) \\ &= -1 + 1 = 0 \end{aligned}$$

3. If $\nabla \cdot \vec{F} = yz\vec{i} + xz\vec{j} + xy\vec{k}$ then find \vec{F}

(AU-2010)

$$\nabla \cdot \vec{F} = yz\vec{i} + xz\vec{j} + xy\vec{k}$$

$$\vec{i} \frac{\partial f}{\partial x} + \vec{j} \frac{\partial f}{\partial y} + \vec{k} \frac{\partial f}{\partial z} = yz\vec{i} + xz\vec{j} + xy\vec{k}$$

Equating the coefficient of $\vec{i}, \vec{j}, \vec{k}$

$$\frac{\partial f}{\partial x} = yz, \quad \frac{\partial f}{\partial y} = xz, \quad \frac{\partial f}{\partial z} = xy$$

$$\begin{aligned} \int \partial f &= \int yz \partial x & \int \partial f &= \int xz \partial y & \int \partial f &= \int xy \partial z \\ f_1 &= xyz + f(y,z) & f_2 &= xyz + f(x,z) & f_1 &= xyz + f(x,y) \\ & & & & & \text{F} = xyz + c \end{aligned}$$

4. Find the unit normal to the surface $x^2 + y^2 - 2z + 3 = 0$ at $(1, 2, -1)$

(AU-2011)

$$\text{Given } \Phi = x^2 + y^2 - 2z + 3 = 0$$

$$\begin{aligned} \nabla \phi &= \left(\frac{\partial \phi}{\partial x} \vec{i} + \frac{\partial \phi}{\partial y} \vec{j} + \frac{\partial \phi}{\partial z} \vec{k} \right) (x^2 + y^2 - 2z + 3) \\ &= 2x\vec{i} + 2y\vec{j} - 2\vec{k} \end{aligned}$$

$$(\nabla \phi)_{(1,2,-1)} = (2\vec{i} + 4\vec{j} - 2\vec{k}), |\nabla \phi| = 2\sqrt{6}$$

$$\hat{n} = \frac{\nabla \phi}{|\nabla \phi|} = \frac{2\vec{i} + 4\vec{j} - 2\vec{k}}{2\sqrt{6}}$$

5. In what direction from $(3,1,-2)$ is the directional derivative of $\phi = x^2y^2z^4$ maximum? Find

Also the magnitude of this maximum.

(AU-2015)

$$\nabla\phi = 2xy^2z^4\vec{i} + 2x^2yz^4\vec{j} + 4x^2y^2z^3\vec{k}$$

$$\text{At}(3,1,-2), \nabla\phi = 96(\vec{i} + 3\vec{j} - 3\vec{k})$$

$$\text{Direction of Maximum} = \nabla\phi = 96(\vec{i} + 3\vec{j} - 3\vec{k})$$

$$\text{Magnitude} = |\nabla\phi| = 96\sqrt{1+9+9} = 96\sqrt{19}$$

6. Prove that $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is irrotational.

(AU-2012)

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix}$$

$$\vec{i}(x-x) - \vec{j}(y-y) + \vec{k}(z-z) = \vec{0}$$

$\therefore \vec{F}$ is irrotational.

7. Find λ so that $\vec{F} = (3x-2y+z)\vec{i} + (4x+\lambda y-z)\vec{j} + (x-y+2k)\vec{k}$ is solenoidal

(AU-2015)-2

Given \vec{F} is solenoidal then $\nabla \cdot \vec{F} = 0$

$$\nabla \cdot \vec{F} = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right) \cdot \left((3x-2y+z)\vec{i} + (4x+\lambda y-z)\vec{j} + (x-y+2k)\vec{k} \right)$$

$$3 + \lambda + 2 = 0$$

$$\lambda = -5$$

8. If \vec{A} and \vec{B} are irrotational, prove that $\vec{A} \times \vec{B}$ is solenoidal.

(AU-2013)

If \vec{A} and \vec{B} are irrotational.

$$\nabla \times \vec{A} = 0, \nabla \times \vec{B} = 0$$

$$\text{We know that } \nabla \cdot (\vec{A} \times \vec{B}) = (\nabla \times \vec{A}) \cdot \vec{B} - (\nabla \times \vec{B}) \cdot \vec{A} = 0 - 0 = 0$$

$\therefore \vec{A} \times \vec{B}$ is solenoidal.

9. Define solenoidal vector function. If $\vec{V} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+2\lambda z)\vec{k}$ is solenoidal, then

find the value of λ

(AU-2013)

$$\text{Given that } \vec{V} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+2\lambda z)\vec{k}$$

$$\nabla \cdot \vec{V} = 0 \text{ if } \vec{V} \text{ is solenoidal}$$

$$\nabla \cdot \vec{V} = \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot (x+3y)\vec{i} + (y-2z)\vec{j} + (x+2\lambda z)\vec{k} = 0$$

$$= \left(\frac{\partial}{\partial x}(x+3y) + \frac{\partial}{\partial y}(y-2z) + \frac{\partial}{\partial z}(x+2\lambda z) \right) = 0$$

$$= 1+1+2\lambda = 0$$

$$\lambda = -1$$

10. Find the value of the constant a, b, c so that the vector

$$\vec{F} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy+2z)\vec{k} \text{ is irrotational}$$

(AU-2010)

$$\begin{aligned}\nabla \times \vec{F} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+2y+az & bx-3y-z & 4x+cy+2z \end{vmatrix} \\ &= \vec{i} \left[\frac{\partial}{\partial x}(4x+cy+2z) - \frac{\partial}{\partial z}(bx-3y-z) \right] - \vec{j} \left[\frac{\partial}{\partial x}(4x+cy+2z) - \frac{\partial}{\partial z}(x+2y+az) \right] \\ &\quad + \vec{k} \left[\frac{\partial}{\partial x}(bx-3y-z) - \frac{\partial}{\partial y}(x+2y+az) \right] \\ &= \vec{i}(c+1) - \vec{j}(4-a) + \vec{k}(b-2)\end{aligned}$$

Given \vec{F} is irrotational, $\nabla \times \vec{F} = 0$.

$$\vec{i}(c+1) - \vec{j}(4-a) + \vec{k}(b-2) = 0$$

Each component should be zero.

$$c+1=0, a-4=0, b-2=0$$

$$c=-1, a=4, b=2.$$

11. Prove that $\nabla \cdot r^n = nr^{n-2} \cdot \vec{r}$

(AU-2011)

$$\text{Let } \vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \quad |\vec{r}| = \sqrt{x^2 + y^2 + z^2},$$

$$\begin{aligned}\nabla \cdot r^n &= \vec{i} \frac{\partial}{\partial x}(r^n) + \vec{j} \frac{\partial}{\partial y}(r^n) + \vec{k} \frac{\partial}{\partial z}(r^n) \\ &= \vec{i} \left[nr^{n-1} \frac{\partial r}{\partial x} \right] + \vec{j} \left[nr^{n-1} \frac{\partial r}{\partial y} \right] + \vec{k} \left[nr^{n-1} \frac{\partial r}{\partial z} \right] \\ &= nr^{n-1} \left[\vec{i} \frac{\partial r}{\partial x} + \vec{j} \frac{\partial r}{\partial y} + \vec{k} \frac{\partial r}{\partial z} \right] \\ &= nr^{n-1} \left[\vec{i} \frac{x}{r} + \vec{j} \frac{y}{r} + \vec{k} \frac{z}{r} \right] \\ &= nr^{n-1} \frac{1}{r} [x\vec{i} + y\vec{j} + z\vec{k}] \\ &= nr^{n-2} \vec{r}\end{aligned}$$

12. Find the directional derivative $\phi = x^2 + y^2 + z^2$ in the direction of the vector

$$\vec{F} = \vec{i} + 2\vec{j} + 2\vec{k} \text{ at } (1,1,1)$$

(AU-2014)

$$\text{Unit normal vector } \hat{n} \text{ in the direction of } \vec{i} + 2\vec{j} + 2\vec{k} \text{ is } \left(\frac{\vec{i} + 2\vec{j} + 2\vec{k}}{3} \right)$$

$$\text{Directional derivative} = \nabla \phi \cdot \hat{n}$$

$$\nabla \phi = \vec{i} \frac{\partial}{\partial x}(x^2 + y^2 + z^2) + \vec{j} \frac{\partial}{\partial y}(x^2 + y^2 + z^2) + \vec{k} \frac{\partial}{\partial z}(x^2 + y^2 + z^2)$$

$$\text{grad } \phi = 2x\vec{i} + 2y\vec{j} + 2z\vec{k}$$

$$\nabla \phi_{at(1,1,1)} = 2\vec{i} + 2\vec{j} + 2\vec{k}$$

$$\text{Directional derivative} = \nabla \phi \cdot \hat{n} = \left(2\vec{i} + 2\vec{j} + 2\vec{k} \right) \cdot \left(\frac{\vec{i} + 2\vec{j} + 2\vec{k}}{3} \right) = \frac{10}{3}$$

13. Find the unit normal vector to the surface $x^2+y^2=z$ at $(1,-2,5)$

(AU-2014)

$$\begin{aligned}\phi &= x^2+y^2-z \\ \nabla\phi &= \left(\frac{\partial\phi}{\partial x} \vec{i} + \frac{\partial\phi}{\partial y} \vec{j} + \frac{\partial\phi}{\partial z} \vec{k} \right) x^2+y^2-z \\ &= 2x\vec{i} + 2y\vec{j} - \vec{k} \\ (\nabla\phi)_{(1,-2,5)} &= (2\vec{i} - 4\vec{j} - \vec{k}), |\nabla\phi| = \sqrt{21} \\ \hat{n} &= \frac{\nabla\phi}{|\nabla\phi|} = \frac{2\vec{i} - 4\vec{j} - \vec{k}}{\sqrt{21}}\end{aligned}$$

14. Show that $\vec{F} = (x^2\vec{i} + y^2\vec{j} + z^2)\vec{k}$ is a conservative vector field.

(AU-2009)

$$\text{If } \hat{F} \text{ is conservative then } \nabla \times \hat{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & y^2 & z^2 \end{vmatrix} = 0\vec{i} + 0\vec{j} + 0\vec{k} = 0$$

Therefore \hat{F} is a conservative vector field.

15. Prove that $\text{curl}(\text{grad } \phi) = 0$

(AU-2014)

$$\begin{aligned}\text{curl}(\text{grad } \phi) &= \text{curl}(\nabla\phi) = \nabla \times \nabla\phi = \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \times \left(\frac{\partial\phi}{\partial x} \vec{i} + \frac{\partial\phi}{\partial y} \vec{j} + \frac{\partial\phi}{\partial z} \vec{k} \right) \\ &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial\phi}{\partial x} & \frac{\partial\phi}{\partial y} & \frac{\partial\phi}{\partial z} \end{vmatrix} = \vec{i} \left[\frac{\partial^2\phi}{\partial y\partial z} - \frac{\partial^2\phi}{\partial y\partial z} \right] - \vec{j} \left[\frac{\partial^2\phi}{\partial x\partial z} - \frac{\partial^2\phi}{\partial x\partial z} \right] + \vec{k} \left[\frac{\partial^2\phi}{\partial y\partial x} - \frac{\partial^2\phi}{\partial y\partial x} \right] = 0\end{aligned}$$

16. Find $\text{Curl } \vec{F}$ if $\vec{F} = xy\vec{i} + yz\vec{j} + zx\vec{k}$

(AU-2014)

$$\text{Curl } \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & zx \end{vmatrix} = \vec{i}(-y) - z\vec{j} - x\vec{k}$$

17. If $\vec{F} = x^2\vec{i} + xy\vec{j}$ evaluate $\int_c \vec{F} \cdot d\vec{r}$ from $(0,0)$ to $(1,1)$ along the line $y = x$.

(AU-2010)

$$\text{Given } \vec{F} = x^2\vec{i} + xy\vec{j}$$

Along the line $y = x$, $dy = dx$

$$\therefore \vec{F} = x^2\vec{i} + x.x\vec{j}, \quad d\vec{r} = dx\vec{i} + dy\vec{j} = dx\vec{i} + dx\vec{j}$$

$$\vec{F} \cdot d\vec{r} = (x^2\vec{i} + x^2\vec{j}) \cdot (dx\vec{i} + dx\vec{j})$$

$$= x^2 dx + x^2 dx = 2x^2 dx$$

$$\int_c \vec{F} \cdot d\vec{r} = \int_0^1 2x^2 dx = \frac{2}{3}$$

18. If $\vec{F} = 5xy\vec{i} + 2y\vec{j}$, evaluate $\int_c \vec{F} \cdot d\vec{r}$ Where C is the part of the curve $y = x^2$ between $x = 1$

and $x = 2$.

(AU-2012)

$$\vec{F} \cdot d\vec{r} = (5xy\vec{i} + 2y\vec{j}) \cdot (dx\vec{i} + dy\vec{j} + dz\vec{k})$$

$$= 5xydx + 2ydy$$

The curve C: $y = x^2$

$$dy = 2xdx$$

x varies from 1 to 2

$$\int_c \vec{F} \cdot d\vec{r} = \int_1^2 5x(x^2)dx + 2x^2 \cdot 2xdx = \left[5\frac{x^4}{4} + \frac{4x^4}{4} \right]_1^2$$

$$= 36 - \frac{9}{4} = \frac{135}{4}$$

19. Find $\iint_S \vec{r} \cdot d\vec{s}$ where S the surface of the tetrahedron whose vertices are is

(0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1).

(AU-2010)

By Gauss divergence theorem

$$\begin{aligned} \iint_S \vec{r} \cdot d\vec{s} &= \iiint_V \nabla \cdot \vec{r} dv \\ &= \iiint_V \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot (x\vec{i} + y\vec{j} + z\vec{k}) dv \\ &= \iiint_V (1+1+1) dv \\ &= \iiint_V 3 dv \\ &= 3 \int_0^1 \int_0^1 \int_0^1 dx dy dz = 3. \end{aligned}$$

20. If S is any closed surface enclosing a volume V and $\vec{F} = ax\vec{i} + by\vec{j} + cz\vec{k}$, prove that

$$\iint_S \vec{F} \cdot \hat{n} ds = (a+b+c)V.$$

(AU-2011)

Gauss Divergence theorem is

$$\begin{aligned} \iint_S \vec{F} \cdot \hat{n} ds &= \iiint_V \nabla \cdot \vec{F} dV \\ &= \iiint_V \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) \cdot (ax\vec{i} + by\vec{j} + cz\vec{k}) dv \\ &= \iiint_V (a+b+c) dv \\ &= (a+b+c)V \end{aligned}$$

21. State Green's theorem in a plane.

(AU-2010)

If $M(x,y)$ and $N(x,y)$ and its partial derivatives are continuous and one valued functions in the region R of the xy plane bounded by a simple closed curve C , then

$$\int_C Mdx + Ndy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy$$

Where C is the curve prescribed in the positive direction.

22. Using Green's theorem, Prove that the area enclosed by a simple closed curve C

$$\text{is } \frac{1}{2} \int (xdy - ydx).$$

(AU-2011)

By Green's theorem

$$\int_C Mdx + Ndy = \iint_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy$$

Let $M = -y$ $N = x$

$$\int_C -ydx + xdy = \iint_S (1+1) dx dy$$

$$= 2 \iint_S dx dy$$

$$= 2(\text{area enclosed by } C)$$

Therefore Area enclosed by $C = \frac{1}{2} \int (xdy - ydx)$

23. State Gauss Divergence theorem.

(AU-2012)

If V is the volume bounded by a closed surface S and if a vector function \vec{F} is continuous and has continuous partial derivatives in V and on S then

$$\iint_S \vec{F} \cdot \hat{n} ds = \iiint_V \nabla \cdot \vec{F} dv$$

24. State Stoke's theorem.

(AU-2015) (2)

The surface integral of the normal component of the curl of a vector function F over an Open surface S is equal to the line integral of the tangential component of F around the

Closed curve C bounding S .
$$\int_C \vec{F} \cdot d\vec{r} = \iint_S \nabla \times \vec{F} \cdot \hat{n} ds$$

PART-B

1. a. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$. Prove that $div(r^n \vec{r}) = (n+3)r^n$ and $curl(r^n \vec{r}) = 0$.

(AU-2011) (8)

b. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then prove that $div grad(r^n) = n(n+1)r^{n-2}$. Hence deduce

$$\text{that } div grad\left(\frac{1}{r}\right) = 0$$

(AU-2015)-2(8)

2. a. Find the directional derivative of $\phi = 3x^2 + 2y - 3z$ at $(1,1,1)$ in the direction of

$$2\vec{i} + 2\vec{j} - \vec{k}$$

(AU-2012) (8)

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.

(AU-2012)(8)

3. a. Find the angle between the normal's to the surfaces $x^2 = yz$ at the points $(1,1,1)$ and $(2,4,1)$

(AU-2014)(8)

b. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $P(1, -2, -1)$ that is maximum and also in the direction of PQ where Q is $(3, -3, -2)$

(AU-2010) (8)

4. a. Evaluate $\int_C \phi d\vec{V}$ where C is the curve $x=t, y=t^2, z=1-t$ and $\phi = x^2y(1+z)$ from

$$t=0 \text{ to } t=1$$

(AU-2011)(8)

b. If $\nabla \phi = (x + 2y + 4z)\vec{i} + (2x - 3y - z)\vec{j} + (4x - y + 2z)\vec{k}$, find the Scalar point function ϕ .

(AU-2011)(8)

5. a. Find the value of n so that the vector $r^n \vec{r}$ is both solenoidal and irrotational

(AU-2015)-2(8)

b. Prove that $\vec{F} = (x^2 - y^2 + x)\vec{i} - (2xy + y)\vec{j}$ is irrotational and hence find its scalar potential.

(AU-2014)(8)

6. a. Prove that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational. Hence find its scalar potential ϕ

(AU-2015)(8)

b. Prove that $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$ is irrotational and hence find its scalar potential.

(AU-2014)(8)

7. a. Find the work done the force $\vec{F} = 3xy\vec{i} - y^3\vec{j}$ moves a particle along the Curve C: $y=2x^2$ from (0, 0) to (1, 2) in the xy-plane. (AU-2011)(8)
- b. Evaluate $\int_C \vec{f} \cdot d\vec{r}$ where $\vec{f} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ along the straight line Joining (1, -2, 1) and (3, 2, 4) (AU-2012)(8)
8. a. Show that $\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is a conservative force field. Find the Scalar potential and the work done by \vec{F} in moving an object in this field from (1, -2, 1) to (3, 1, 4) (AU-2009)(8)
- b. Find the directional derivative of $xy^2 + yz^3$ at (2, -1, 1) in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at (-1, 2, 1) (AU-2009)(8)
9. a. Evaluate $\iint_S \vec{f} \cdot \hat{n} ds$ Where $\vec{f} = (x^2 + y^2)\vec{i} - 2xz\vec{j} + 2yz\vec{k}$ and S is the surface of the $2x + y + 2z = 6$ in the first octant. (AU-2010)(8)
- b. Using Green's theorem in the plane evaluate $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ Where C is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$. (AU-2009)(8)
10. a. Using Green's theorem ,evaluate $\int_C (y - \sin x)dx + \cos x dy$ where C is the triangle formed by $y=0, x = \frac{\pi}{2}, y = \frac{2x}{\pi}$ (AU-2015)(8)
- b. Apply Green's theorem in the plane to evaluate $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ Where C is the boundary of the region defined by $x = 0, y = 0$ and $x + y = 1$. (AU-2014)-2(8)
11. a. Verify Green's theorem in a plane for $\int (xy + y^2)dx + x^2 dy$ where C is the boundary of the common area between $y = x^2$ and $y=x$ in the xoy plane (AU-2014)(8)
- b. Using Green's theorem ,evaluate $\int_C (x^2 - 2xy)dx + (x^2 y + 3)dy$,where C is the region bounded by the curves $y^2=8x$ and $x=2$ (AU-2015)(8)
12. a. Verify Gauss Divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ Where S is the surface of the Cuboid formed by the planes $x = 0, x = a, y = 0, y = b, z = 0, z = c$. (AU-2014)(8)
- b. Verify Gauss's divergence theorem for the function $\vec{F} = y\vec{i} + x\vec{j} + z^2\vec{k}$ Over the cylindrical region bounded by $x^2 + y^2 = 9, z = 0$ and $z = 2$. (AU-2012)(8)
13. Verify Gauss's divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ and C is it's boundary over the cube $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (AU-2015)-3(16)
14. Verify Gauss Divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ taken over the cube bounded by the Planes $x=0, y=0, z=0, x=1, y=1$ and $z=1$ (AU-2015)(16)
15. a. Verify stoke's theorem for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ in the rectangular region in the xy plane bounded by the lines $x = 0, x = a, y = 0, y = b$. (AU-2015)-3(8)
- b. Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ where S is the surface of upper hemisphere $x^2 + y^2 + z^2 = 1$ and C is its boundary in xy-plane. (AU-2014)(8)

UNIT-V
LAPLACE TRANSFORMS

PART-A

1. State the sufficient condition for existence of the Laplace transform of f(t)

(AU-2015)

(i) f(t) should be continuous or piecewise continuous in the given closed interval [a,b] where a>0

(ii) f(t) should be of exponential order.

2. Find the Laplace transform of $f(t) = t \cosh t$

(AU-2014)

$$\begin{aligned} L(t \cosh t) &= \frac{-d}{ds} (L(\cosh t)) = \frac{-d}{ds} \left[\frac{s}{s^2 - 1} \right] \\ &= - \left[\frac{(s^2 - 1)(1) - s(2s)}{(s^2 - 1)^2} \right] = - \left[\frac{-1 - s^2}{(s^2 - 1)^2} \right] = \frac{s^2 + 1}{(s^2 - 1)^2} \end{aligned}$$

3. Find the Laplace transform of $\frac{t}{e^t}$

(AU-2013)

$$L\left(\frac{t}{e^t}\right) = L(te^{-t}) = L(t)_{s \rightarrow s+1} = \left(\frac{1}{s^2}\right)_{s \rightarrow s+1} = \frac{1}{(s+1)^2}$$

4. State and prove change of scale property in Laplace transform.

(AU-2012)

$$\text{If } L(f(t)) = F(s), \text{ then } L(f(at)) = \frac{1}{a} F\left(\frac{s}{a}\right).$$

$$L(f(at)) = \int_0^{\infty} e^{-st} f(at) dt$$

$$\begin{aligned} at &= u & t &= 0 & u &= 0 \\ adt &= du & t &= \infty & u &= \infty \end{aligned}$$

$$L(f(at)) = \int_0^{\infty} e^{-\frac{su}{a}} f(u) \frac{du}{a}$$

$$= \frac{1}{a} \int_0^{\infty} e^{-\frac{su}{a}} f(u) du$$

$$L(f(at)) = \frac{1}{a} F\left(\frac{s}{a}\right)$$

5. State the first shifting theorem on Laplace transforms.

(AU-2012)

If $L(f(t)) = F(s)$ then $L[e^{at}f(t)] = F[s-a]$ and

If $L(f(t)) = F(s)$ then $L[e^{-at}f(t)] = F[s+a]$

6. Find the Laplace transform of $\frac{e^{-2t}}{\sqrt{t}}$

(AU-2012)

$$L[t^{-1/2} e^{-2t}] = L[t^{-1/2}]_{s \rightarrow s+2}$$

$$= \left[\frac{\sqrt{\pi}}{s} \right]_{s \rightarrow s+2}$$

$$= \left[\frac{\sqrt{\pi}}{s+2} \right]$$

7. Find the Laplace transform of $\sqrt{t}e^{3t}$

(AU-2012)

$$L[t^{1/2} e^{3t}] = L[t^{1/2}]_{s \rightarrow s-3}$$

$$= \left[\frac{\sqrt{\pi}}{2s^{3/2}} \right]_{s \rightarrow s-3}$$

$$= \frac{\sqrt{\pi}}{2(s-3)^{3/2}}$$

8. Find $L[\cos^2 3t]$ (AU-2011)

$$\begin{aligned} L[\cos^2 3t] &= L\left[\frac{1 + \cos 6t}{2}\right] \\ &= \frac{1}{2}L[1 + \cos 6t] \\ &= \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2 + 36}\right] \end{aligned}$$

9. Find $L[(t^3 + 3e^{2t} - 5\sin 3t)e^{-t}]$ (AU-2011)

$$\begin{aligned} L[(t^3 + 3e^{2t} - 5\sin 3t)e^{-t}] &= L[(t^3 e^{-t} + 3e^{2t} e^{-t} - 5\sin 3t e^{-t})] \\ &= L[(t^3 e^{-t} + 3e^t - 5\sin 3t e^{-t})] \\ &= L[t^3]_{s \rightarrow s+1} + 3L(1)_{s \rightarrow s+1} - 5L[\sin 3t]_{s \rightarrow s+1} \\ &= \left[\frac{6}{s^4} + \frac{3}{s} - \frac{3}{s^2 + 9}\right]_{s \rightarrow s+1} \\ &= \left[\frac{6}{(s+1)^4} + \frac{3}{s+1} - \frac{3}{(s+1)^2 + 9}\right]_{s \rightarrow s+1} \end{aligned}$$

10. Find $L\left[\frac{\sin t}{t}\right]$ (AU-2014)

$$\begin{aligned} L\left[\frac{\sin t}{t}\right] &= \int_s^\infty L(\sin t) ds = \int_s^\infty \frac{1}{s^2 + 1} ds = [\tan^{-1}(s)]_s^\infty = [\tan^{-1}(\infty) - \tan^{-1}(s)] \\ &= \frac{\pi}{2} - \tan^{-1}(s) = \cot^{-1}(s) \end{aligned}$$

11. Find the Laplace transform of the function $f(t) = \frac{1 - e^{-t}}{t}$ (AU-2013)

$$\begin{aligned} L(f(t)) &= L\left[\frac{1 - e^{-t}}{t}\right] = \int_s^\infty L(1 - e^{-t}) ds = \int_s^\infty L(1) - L(e^{-t}) ds = \int_s^\infty \left[\frac{1}{s} - \frac{1}{s+1}\right] ds \\ &= [\log s - \log(s+1)]_s^\infty \\ &= \log\left[\frac{s+1}{s}\right] \end{aligned}$$

12. Verify initial value theorem for $f(t) = 1 + e^{-t}(\sin t + \cos t)$ (AU-2012)

Initial value theorem is, if $L[f(t)] = F(s)$, then $\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} sF(s)$

$$F(s) = L(1 + e^{-t}(\sin t + \cos t))$$

$$F(s) = \frac{1}{s} + L[\sin t + \cos t]_{s \rightarrow s+1}$$

$$F(s) = \frac{1}{s} + \frac{s+2}{(s+1)^2 + 1}$$

$$\lim_{t \rightarrow 0} f(t) = \lim_{t \rightarrow 0} 1 + e^{-t}(\sin t + \cos t) = 2$$

$$\lim_{s \rightarrow \infty} sF(s) = \lim_{s \rightarrow \infty} s \left[\frac{1}{s} + \frac{s+2}{(s+1)^2 + 1} \right] = \lim_{s \rightarrow \infty} \left[1 + \frac{s^2 \left(1 + \frac{2}{s} \right)}{s^2 \left(1 + \frac{2}{s} + \frac{2}{s^2} \right)} \right] = 2$$

$$\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} sF(s) = 2$$

Hence the initial value theorem is verified.

13. Verify Initial value theorem for the function $f(t) = ae^{-bt}$

(AU-2013)

$$f(t) = ae^{-bt}, F(s) = L[f(t)] = L[ae^{-bt}] = \frac{a}{s+b}$$

$$\text{Initial value theorem: } \lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} sF(s)$$

$$\text{L.H.S} = \lim_{t \rightarrow 0} f(t) = \lim_{t \rightarrow 0} ae^{-bt} = a$$

$$\begin{aligned} \text{R.H.S } \lim_{s \rightarrow \infty} sF(s) &= \lim_{s \rightarrow \infty} s \left(\frac{a}{s+b} \right) = \lim_{s \rightarrow \infty} \frac{as}{s+b} \\ &= \lim_{s \rightarrow \infty} \frac{as}{\left(1 + \frac{b}{s} \right)} = \lim_{s \rightarrow \infty} \frac{a}{1 + \left(\frac{b}{s} \right)} = a \end{aligned}$$

Hence the initial value theorem is verified.

14. If $L(e^{-t} \cos^2 t) = F(s)$, find $\lim_{s \rightarrow 0} sF(s)$

(AU-2013)

$$\begin{aligned} F(s) = L(e^{-t} \cos^2 t) &= L[\cos^2 t]_{s \rightarrow s+1} \\ &= L \left[\frac{1 + \cos 2t}{2} \right]_{s \rightarrow s+1} \\ &= \frac{1}{2} L[1 + \cos 2t]_{s \rightarrow s+1} = \frac{1}{2} \left[\frac{1}{s} + \frac{s}{s^2 + 4} \right]_{s \rightarrow s+1} \end{aligned}$$

$$L(e^{-t} \cos^2 t) = \frac{1}{2} \left[\frac{1}{s+1} + \frac{s}{(s+1)^2 + 4} \right]$$

15. Define periodic function with an example.

(AU-2010)

A function $f(t)$ is said to have a period T or to be periodic with period T if for all t , $f(t+T) = f(t)$ where T is a positive constant. The least value of $T > 0$ is called the period of $f(t)$.

$$f(t) = \sin t$$

$$f(t + 2\pi) = \sin(t + 2\pi)$$

$$\text{Eg. Consider } = \sin t$$

$$\text{i.e. } f(t) = f(t + 2\pi) = \sin t$$

$\therefore \sin t$ is a periodic function with period 2π

16. Evaluate $L^{-1} \left[\frac{1}{s^2 + 6s + 13} \right]$

(AU-2014)

$$L^{-1}\left[\frac{1}{s^2 + 6s + 13}\right] = L^{-1}\left[\frac{1}{s^2 + 6s + 9 + 4}\right] = L^{-1}\left[\frac{1}{(s+3)^2 + 2^2}\right] = e^{-3t} \frac{\sin 2t}{2}$$

17. Find the Laplace inverse transform of $\frac{1}{(s+1)(s+2)}$ (AU-2012)

$$\frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

A=1 and B=1 (using partial fraction)

$$L^{-1}\left[\frac{1}{(s+1)(s+2)}\right] = L^{-1}\left[\frac{1}{s+1}\right] + L^{-1}\left[\frac{1}{s+2}\right]$$

$$L^{-1}\left[\frac{1}{(s+1)(s+2)}\right] = e^{-t} + e^{-2t}$$

18. Find the inverse Laplace transform of $\log\left(\frac{s+1}{s-1}\right)$ (AU-2012)

We know that $L^{-1}[F(s)] = -\frac{1}{t} L^{-1}[F'(s)]$

$$L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right] = -\frac{1}{t} L^{-1}\left[\frac{d}{ds}\left[\log\left(\frac{s+1}{s-1}\right)\right]\right]$$

$$= -\frac{1}{t} L^{-1}\left[\frac{d}{ds}[\log(s+1) - \log(s-1)]\right]$$

$$= -\frac{1}{t} L^{-1}\left[\frac{1}{s+1} - \frac{1}{s-1}\right] = -\frac{1}{t} [e^{-t} - e^t] = \frac{2}{t} \sinh t$$

19. Find the Laplace transform of $\int_0^t te^{-t} dt$ (AU-2015)

$$\begin{aligned} L\left[\int_0^t te^{-t} dt\right] &= \frac{1}{s} L[te^{-t}] \\ &= \frac{1}{s} \left[-\frac{d}{ds} L[e^{-t}]\right] \\ &= \frac{1}{s} \left[-\frac{d}{ds} \left(\frac{1}{s+1}\right)\right] \\ &= \frac{-1}{s} \left(\frac{1}{(s+1)^2}\right) = \frac{-1}{s(s+1)^2} \end{aligned}$$

20. Find the inverse Laplace transform of $\frac{e^{-\pi s}}{(s-1)^2}$ (AU-2014)

$$L^{-1}\left(\frac{1}{s^2}\right) = t \quad \text{and} \quad L^{-1}\left(\frac{1}{(s-1)^2}\right) = te^t$$

$$L\left(\frac{e^{-\pi s}}{(s-1)^2}\right) = (t - \pi)e^{(t-\pi)}$$

PART-B

1. a. Find $L(t^2 e^{-3t} \sin 2t)$ (AU-2013)(8)

b. Find $L\left[\frac{\cos at - \cos bt}{t}\right]$ (AU-2015)(8)

2. a. Find the Laplace transform of the square-wave function (or Meander function) of

Period a defined as $f(t) = \begin{cases} 1, & \text{when } 0 < t < \frac{a}{2} \\ -1, & \text{when } \frac{a}{2} < t < a \end{cases}$ (AU-2013)(8)

b. Find the Laplace transform of the following triangular wave function given by

$f(t) = \begin{cases} t & 0 \leq t \leq c \\ 2c - t & c \leq t \leq 2c \end{cases}$ and $f(t + 2c) = f(t)$. (AU-2015)(8)

3. a. Find the Laplace transform of the periodic function defined on the interval by $0 \leq t \leq 1$

$f(t) = \begin{cases} -1, & 0 \leq t \leq \frac{1}{2} \\ 1 & \frac{1}{2} \leq t < 1 \end{cases}$ and $f(t + 1) = f(t)$. (AU-2014)(8)

b. Find the Laplace transform of $f(t) = t^2$, $0 < t < 2$ and $f(t + 2) = f(t)$ for all $t > 0$ (AU-2013)(8)

4. a. Find the inverse Laplace transform of $\frac{4s + 7}{s^2(2s + 3)(3s + 5)}$ (AU-2013)(8)

b. Find $L^{-1}(s/(s^2+1)(s^2+4))$ (AU-2015)(8)

5.a. Find the Laplace transforms of the following functions 1) $e^t \cos t$ 2) $1 - \cos t$ π/w (AU-2014)(8)

b. Find the Laplace transform of $f(t) = te^{-3t} \cos 2t$. (AU-2014)(8)

6. a. Find the Laplace transforms of $f(t) = \begin{cases} t & 0 < t < a \\ 2a - t & a < t < 2a \end{cases}$,

where $f(t + 2a) = f(t)$ (AU-2014)(8)

b. Find the Laplace transform of $f(t)$ where

$f(t) = \begin{cases} \sin \omega t & \text{for } 0 < t < \frac{\pi}{\omega} \\ 0 & \text{for } \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$, $f\left(t + \frac{2\pi}{\omega}\right) = f(t)$ (AU-2014)(8)

7. a. Find the Laplace transform $f(t) = \begin{cases} \cos t & 0 < t < \pi \\ 0 & \pi < t < 2\pi \end{cases}$, $f(t + 2\pi) = f(t)$

b. Find $L^{-1}\left[\frac{3s^2 + 16s + 26}{s(s^2 + 4s + 13)^2}\right]$ (AU-2013)(8)

8.a. Find $L^{-1}\left[\frac{s^2}{(s^2 + a^2)^2}\right]$ and find $L^{-1}\left[\frac{1}{(s^2 + a^2)^2}\right]$ hence find $L^{-1}\left[\frac{1}{(s^2 + 9s + 13)^2}\right]$ (AU-2013)(8)

b. Use convolution theorem to find the inverse of $\frac{s}{(s^2 + 4)(s^2 + 9)}$ (AU-2013)(8)

9. a. Find the Laplace transform of $f(t) = \frac{\cosh t \cos t}{t}$ and $g(t) = \sin \sqrt{t}$ (AU-2013)(8)

b. Using convolution theorem to find the inverse Laplace transform of the function

$$\frac{s}{(s^2 + 1)^2}$$

(AU-2014)(8)

10.a. Using convolution theorem to find the inverse Laplace transform of the function

$$\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$$

(AU-2014)-2(8)

b. Using convolution, solve the initial value problem, $y'' + 9y = \sin 3t$,
 $y(0) = 0, y'(0) = 0$.

11. a. Verify initial and final value theorem for $f(t) = 1 + e^{-t}(\sin t + \cos t)$ (AU-2014)(8)

b. Verify initial and final value theorem for $f(t) = 1 + e^{-2t}$

12.a. Solve $y'' + y' = t^2 + 2t$, $y(0) = 4$, $y'(0) = -2$ by using Laplace transform. (AU-2013)(8)

b. Solve the differential equation $y'' - 3y' + 2y = 4t + e^{3t}$ where $y(0) = 1$ and $y'(0) = -1$

using Laplace transforms

(AU-2015)(8)

13.a Solve $y'' - 6y' + 9y = t^2 e^{3t}$, $y(0) = 2$, $y'(0) = 6$ by Laplace transform method (AU-2014)(8)

b. Solve the following differential equation, using Laplace transform

$$y'' + 2y' + 2y = 8e^t \sin t, y(0) = y'(0) = 0$$

(AU-2013)(8)

14.a. Using Laplace Transform, solve $\frac{d^2 y}{dt^2} + 4y = \sin 2t$ given $y(0) = 3$, $y'(0) = 4$ (AU-2014)(8)

b. Use Laplace transform to solve $(D^2 - 3D + 2)y = e^{3t}$ with $y(0) = 1$, $y'(0) = 0$ (AU-2014)(8)
 Laplace transform method, solve $d^2 y/dt^2 + 9y = 18t$ given that
 $y(0) = 0, y(\pi/2) = 0$.

15.a. Using

b. Using Laplace transform, find the solution of $y' + 3y + 2 \int_0^t y dt = t$ $y(0) = 0$

UNIT-III
ANALYTIC FUNCTIONS
PART- A

1. State the basic difference between the limit of a function of a real variable and that of a complex variable. (AU2012)

Real variable	Complex Variable
Limit takes along x axis and y axis or parallel to both axis	Limit takes along any path (straight or curved)

2. State the necessary condition of Cauchy-Riemann equations (AU-2011)

The necessary condition for $f(z) = u(x, y) + iv(x, y)$ to be analytic in a region R are

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

3. Write 2-D Laplace equations in polar coordinates.

(AU-2011)

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} = 0$$

4. Show that the function $f(z) = \bar{z}$ is nowhere differentiable.

(AU-2014)-2

Given $f(z) = \bar{z} = x - iy$

$$u = x, v = -y$$

$$\frac{\partial u}{\partial x} = 1, \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial y} = -1$$

$u_x \neq v_y$, C-R equations are not satisfied anywhere. Hence

$f(z) = \bar{z}$ is nowhere differentiable.

5. Find the constants a, b if $f(z) = x + 2ay + i(3x + by)$ is analytic

(AU-2013)

$$f(x) = x + 2ay + i(3x + by)$$

$$u = x + 2ay \text{ and } v = (3x + by)$$

Where $\frac{\partial u}{\partial x} = 1, \frac{\partial u}{\partial y} = 2a$

$$\frac{\partial v}{\partial x} = 3, \frac{\partial v}{\partial y} = b$$

We know that by CR equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$a = \frac{3}{2}, \quad b = 1$$

6. If $u+iv$ is analytic, show that $v -iu$ & $-v +iu$ are also analytic

(AU-2013)

Given $u+iv$ is analytic

C-R equations are satisfied $u_x = v_y$ (1)

$$u_y = -v_x \text{(2)}$$

Since the derivatives of u & v exist it is therefore continuous

Now to prove $v -iu$ & $-v +iu$ are also analytic, we should prove that

(i) $v_x = -u_y$ & $v_y = u_x$ &

(ii) $v_x = u_v$ & $v_y = u_x$

(iii) u_x, u_y, v_x, v_y are all continuous. Results (i) & (ii) follows from (1) & (2). since the

derivatives of u & v exist from (1) and (2), the derivatives of u and v should be continuous. Hence the result

7. Find the value of a, b, c, d so that the function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$

may be analytic

(AU-2013)

$$f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$$

$$u = x^2 + axy + by^2, v = cx^2 + dxy + y^2$$

$$u_x = 2x + ay, v_x = 2cx + dy$$

$$u_y = ax + 2by, v_y = dx + 2y$$

$f(z)$ is analytic, $u_x = v_y$ and $u_y = -v_x$

$$a = 2, b = -1, c = -1, d = 2$$

8. State whether or not $f(z) = \bar{z}$ is an analytic function

(AU-2012)-2

$$w = f(z) = \bar{z}$$

$$u + iv = x - iy \Rightarrow u = x \text{ and } v = y$$

$$u_x = 1, v_x = 0$$

$$u_y = 0, v_y = -1$$

$$u_x \neq v_y$$

Hence CR equations are not satisfied

\therefore The function $f(z)$ is nowhere analytic

9. Verify whether or not $f(z) = e^x(\cos y - i \sin y)$ is analytic

(AU-2014)

$$u = e^x \cos y \text{ and } v = -e^x \sin y$$

$$\frac{\partial u}{\partial x} = e^x \cos y \text{ and } \frac{\partial v}{\partial x} = -e^x \sin y$$

$$\frac{\partial u}{\partial y} = -e^x \sin y \text{ and } \frac{\partial v}{\partial y} = -e^x \cos y$$

$$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} \neq -\frac{\partial v}{\partial x}$$

CR equations are not satisfied. It is not an analytic function.

10. S.T $f(z) = e^x \sin y$ is harmonic

(AU-2014)

$$u_x = e^x \sin y, u_y = e^x \cos y$$

$$u_{xx} = e^x \sin y, u_{yy} = -e^x \sin y$$

$$u_{xx} + u_{yy} = e^x \sin y - e^x \sin y = 0$$

$f(z) = e^x \sin y$ is harmonic

11. If $f(z)$ is an analytic function whose real part is constant, Prove that $f(z)$ is a constant function.

(AU-2012)

Let $f(z) = u(x, y) + iv(x, y)$ be an analytic function

$$\text{Therefore by CR equations } \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

Given $u = \text{constant}$

To prove $f(z)$ is a constant

$$u = c$$

$$\frac{\partial u}{\partial x} = 0, \frac{\partial u}{\partial y} = 0$$

$$\text{By CR equation } \frac{\partial u}{\partial x} = 0 \Rightarrow \frac{\partial v}{\partial y} = 0 \text{ and } \frac{\partial v}{\partial y} = 0 \Rightarrow \frac{\partial v}{\partial x} = 0$$

$$\therefore f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} = 0 + i0$$

$$f'(z) = 0 \Rightarrow f(z) = c$$

$f(z)$ is a constant.

12. Find the image of the line $x=k$ under the transformation $w = \frac{1}{z}$ (AU-2013)

Given $w = \frac{1}{z}$

i.e., $z = \frac{1}{w} = \frac{1}{u+iv} = \frac{u-iv}{(u+iv)(u-iv)} = \frac{u-iv}{u^2+v^2}$

$$x+iy = \frac{u}{u^2+v^2} + i\left(\frac{-v}{u^2+v^2}\right)$$

i.e., $x = \frac{u}{u^2+v^2}$ (1), $y = \frac{-v}{u^2+v^2}$ (2)

Given $x=k$ in the z plane

$$k = \frac{u}{u^2+v^2} \text{ by (1),}$$

$$k(u^2+v^2) = u$$

$$u^2+v^2 - \frac{1}{k}u = 0$$

$$\left(u - \frac{1}{2k}\right)^2 + v^2 - \frac{1}{4k^2} = 0$$

$$\left(u - \frac{1}{2k}\right)^2 + v^2 = \frac{1}{4k^2} \text{ which is a circle whose centre is } \left(\frac{1}{2k}, 0\right) \text{ and radius } \frac{1}{2k}$$

13. Find the map of the circle $|z| = 3$ under the transformation $w = 2z$ (AU-2012)

$$w = 2z$$

$$u+iv = 2(x+iy)$$

$$u = 2x, v = 2y \Rightarrow x = \frac{u}{2}, y = \frac{v}{2}$$

Given $|z| = 3 \Rightarrow |x+iy| = 3 \Rightarrow x^2 + y^2 = 9$

$$\therefore \left(\frac{u}{2}\right)^2 + \left(\frac{v}{2}\right)^2 = 9 \Rightarrow u^2 + v^2 = 36$$

Hence the image of $|z| = 3$ in the z -plane is transformed into

$$u^2 + v^2 = 36 \text{ in the } w\text{-plane under the transformation } w = 2z$$

14. Find the image of the circle $|z| = 2$ under the transformation $w = 3z$ (AU-2012)

$$w = 3z$$

$$u+iv = 3(x+iy)$$

$$u = 3x, v = 3y \Rightarrow x = \frac{u}{3}, y = \frac{v}{3}$$

Given $|z| = 2 \Rightarrow |x+iy| = 2 \Rightarrow x^2 + y^2 = 4$

$$\therefore \left(\frac{u}{3}\right)^2 + \left(\frac{v}{3}\right)^2 = 4 \Rightarrow u^2 + v^2 = 36$$

Hence the image of $|z| = 2$ in the z-plane is transformed into

$$u^2 + v^2 = 36 \text{ in the } w\text{-plane under the transformation } w = 3z$$

15. Find the image of the circle $|z| = \lambda$ under the transformation $w = 5z$ (AU-2011)

$$w = 5z$$

$$u + iv = 5(x + iy)$$

$$u = 5x, v = 5y \Rightarrow x = \frac{u}{5}, y = \frac{v}{5}$$

$$\text{Given } |z| = \lambda \Rightarrow |x + iy| = \lambda \Rightarrow x^2 + y^2 = \lambda^2$$

$$\therefore \left(\frac{u}{5}\right)^2 + \left(\frac{v}{5}\right)^2 = \lambda^2 \Rightarrow u^2 + v^2 = (5\lambda)^2$$

Hence the image of $|z| = \lambda$ in the z-plane is transformed into

$$u^2 + v^2 = (5\lambda)^2 \text{ in the } w\text{-plane under the transformation } w = 5z$$

16. Define critical point of a transformation (AU-2010)

A point z_0 at which the mapping $w=f(z)$ is not conformal is called the critical point.

17. Find the invariant points of the transformation $f(z) = z^2$ (AU-2014)

$$f(z) = z^2$$

$$w = z^2,$$

$$z = z^2$$

$$z^2 - z = 0$$

$$z(z - 1) = 0$$

$$z = 0, z = 1$$

The invariant points are $z=0, z=1$.

18. Find the critical points of the transformation $w = 1 + \frac{2}{z}$ (AU-2013)

$$z = 1 + \frac{2}{z} \quad z^2 - z - 2 = 0 \quad (z - 2)(z + 1) = 0$$

$$z = 2, z = -1$$

Critical points are $z=2, -1$

19. Find the invariant points of the transformation $\omega = \frac{2z + 6}{z + 7}$ (AU-2013)

$$\text{The invariant points are given by } z = \frac{2z + 6}{z + 7}$$

$$z^2 + 7z - 2z - 6 = 0 \Rightarrow z^2 + 5z - 6 = 0$$

$$(z + 6)(z - 1) = 0$$

$$z = -6, 1$$

20. Prove that a bilinear transformation has atmost two fixed points. (AU-2012)

$$\text{The fixed points of the transformation } w = \frac{az + b}{cz + d} \text{ is obtained from } z = \frac{az + b}{cz + d}$$

$$\text{or } cz^2 + (d - a)z - b = 0$$

These points are two in number unless the discriminant is zero in which case the number of points is one.

21. Show that $|Z|^2$ is not analytic at any point. (AU-2015)

$$f(z) = |Z|^2 = u + iv = x^2 + y^2 \text{ where } u = x^2 + y^2 \text{ and } v = 0, u_x = 2x, u_y = 2y, v_x = 0, v_y = 0$$

hence $f(z)$ is not analytic.

PART-B

1. a. If $f(z)$ is an analytic function of z , prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ (AU-2013)(8)

b. show that a harmonic function u satisfies the formal differential equation

$$\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0 \quad \text{and hence P.T } \log|f^1(z)| \text{ is harmonic, where } f(z) \text{ is a regular function.}$$

(AU-2015)(8)

2. a. Show that the function $u = e^{-x}(x \cos y + y \sin y)$ is harmonic function.

Hence find the corresponding analytic function $f(z) = u + iv$ (AU-2014)(8)

b. Determine the analytic function $w = u + iv$ given that $u = e^{-x}(x \cos y + y \sin y)$ (AU-2015)(8) 3. a Prove that $u = e^y \cos x$ and $v = e^{-x} \sin y$ satisfy Laplace equations but that $u + iv$ is not an analytic function of z .

b. Find if $\Phi = (x - y)(x^2 + 4xy + y^2)$ can represent the equipotential surface for an electric field. Find the corresponding complex potential $\omega = \phi + i\psi$ and also ψ

(AU-2013)(8) 4.a. Find the analytic function

$f(z) = u + iv$ where $v = 3r^2 \sin 2\theta - 2r \sin \theta$.

Verify that u is a harmonic function.

(AU-2013)(8)

b. Determine the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$ (AU-2014) (8)

5.a. Prove that the function $u = e^x(x \cos y - y \sin y)$ satisfies Laplace's equation and find the corresponding analytic function $f(z) = u + iv$ (AU-2013)(8)

b. Prove that the real and imaginary parts of an analytic function are harmonic function. (AU-2014) (8)

6.a. Find the analytic function $w = u + iv$ if $e^{2x}(x \cos 2y - y \sin 2y)$ and hence find u (AU-2013)(8)

b. Find the analytic function $f(z) = u(x, y) + iv(x, y)$ given that

$$u - v = e^x(\cos y - \sin y) \quad \text{(AU-2014)(8)}$$

7. a. If $u = x^2 - y^2$ and $v = \frac{-y}{x^2 + y^2}$ prove that both u and v satisfy Laplace equations, but

$u + iv$ is not a regular function of z (AU-2013)(8)

b. Find the image of the circle $|z| = 2$ under the transformation $\omega = z + 3 + 2i$ (AU-2013)(8)

8.a. Find the image of w plane of the region of the z -plane bounded by the straight line $x=1, y=1$ and $x+y=1$ under the transformation $w = z^2$ (AU-2013)(8)

b. Find the image in the w -plane of the infinite strip $1/4 \leq y \leq 1/2$ under the transformation $w=1/z$ (AU-2015)(8)

9.a. Prove that $w = \frac{z}{1-z}$ maps the upper half of the z -plane to the upper half of the w -plane and

also find the image of the unit circle of the z plane. (AU-2013)(8)

b. Find the image of the circle $|z - 3i| = 3$ and the region $1 < x < 2$ under the map $w = \frac{1}{z}$

10. a. Find the image of $|z+2i|=2$ under the transformation $w=1/z$.

b. Find the image of the following regions under the transformation $w=1/z$.

i) the half plane $x > c$ when $c > 0$

ii) the half plane $y > c$ when $c < 0$

11.a. S.T under the mapping $w = i-z/i+z$, the image of the circle $x^2 + y^2 < 1$ is the entire half of the w -plane to the right of the imaginary axis

b. Find the image of the region bounded by the lines $x=0, y=0,$ and $x+y=1$ under the

mappings $w = e^{\frac{i\pi}{4}}$ and $w = z + (2 + 3i)$ (AU-2014)(8)

12.a Show that the image of the hyperbola $x^2 - y^2 = 1$ under the transformation

$$w = \frac{1}{z} \text{ is the lemniscates } r^2 = \cos 2\theta$$

(AU-2012)(8)

b. Find the Bilinear transformation which maps $z=0, z=1, z=\infty$ into the points $w=i, w=1, w=-i$

(AU-2013)(8)

13.a. Find the bilinear transformation that maps 1, i, and -1 of the z-plane onto 0, 1 and ∞ on the w- plane. Also find the image of the unit circle of the z plane.

(AU-2014) (8)

b. Find the Bilinear transformation that maps the points $z=\infty, 1, 0$ onto the points $w=0, i, \infty$ respectively

(AU-2012)(8)

14.a. Find the Bilinear transformation that maps the points $z=1, i, -1$ into the points $w=0, 1, \infty$ respectively. Find also the pre-image of $|w|=1$ under this bilinear transformation.

(AU-2014)(8)

b. Find the bilinear transformation that maps the points $z=0, -1, i$ into the points $w=i, 0, \infty$ respectively.

(AU-2015)(8)

15. a. Find the bilinear transformation that maps the points $1+i, -i, 2-i$ of the z- plane into the points $0, 1, i$ of the w-plane.

b. Find the bilinear transformation that maps the points $z=i, -1, 1$ into the points $w=0, 1, \infty$ respectively.

UNIT-IV
COMPLEX INTEGRATION
PART-A

1. State Cauchy's integral theorem

(AU-2015)

If $f(z)$ is analytic inside and on a closed curve c of a simply connected region R and if 'a' is any point within c , then $f(a) = \frac{1}{2\pi i} \int_c \frac{f(z)}{z-a} dz$, the integration around C being taken in the positive direction .

2. Evaluate $\int_C \frac{e^{-z}}{z^2} dz$, where C is a circle $|z|=1$.

(AU-2012)

We know that $\int_c \frac{f(z)}{z-a} dz = 2\pi i f(a)$, $\int_c \frac{e^{-z}}{z^2} dz = \int_c \frac{e^{-z}}{(z-0)^2}$

Here $f(z) = e^{-z}$, $a=0$ is lies inside $|z|=1$

By cauchy's integral formula we get

$$\int_c \frac{e^{-z}}{z^2} dz = 2\pi i f'(a) = 2\pi i(-1) = -2\pi i$$

3. Evaluate $\int_c \frac{z^2 + 1}{z^2 - 1} dz$ where C is a circle of unit radius and centre at $z=i$. (AU-2013)

$$|z - i| = 1$$

The poles $z=1, z=-1$ lies outside the circle

$\therefore \frac{z^2 + 1}{z^2 - 1}$ is analytic inside $|z - i| = 1$

By Cauchy's theorem, $\int_c \frac{z^2 + 1}{z^2 - 1} dz = 0$

4. Evaluate $\int_z \sec z dz$ where c is the unit circle $|z|=1$ (AU-2014)

$$\int_z \sec z dz = \int_z \frac{1}{\cos z} dz$$

The pole are given by the solution of $\cos z = 0$

$$i.e., z = (2n + 1) \frac{\pi}{2}, n = 0, 1, 2, \dots$$

$$z = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

Hence all the poles lies outside $|z|=1$, $\sec z$ is analytic with $|z|=1$

By Cauchy's theorem $\int_z \sec z dz = 0$

5. Evaluate $\int_c \frac{3z^2 + 7z + 1}{(z + 1)} dz$ where C is $|z|=1/2$ (AU-2013)

$$\int_c \frac{3z^2 + 7z + 1}{z - (-1)} dz \quad \text{Here } z=-1 \text{ lies outside c.}$$

$\therefore f(z)$ is analytic inside and on c

$\therefore f'(z)$ is continuous inside c.

Hence by cauchy's theorem $\int_c f(z) dz = 0$

6. State Taylor's theorem. (AU-2011)

A function $f(z)$, is analytic inside a circle C with centre at a, can be expanded in the series

$$f(z) = f(a) + f'(a)(z-a) + \frac{f''(a)}{2!}(z-a)^2 + \frac{f'''(a)}{3!}(z-a)^3 + \dots + \frac{f^{(n)}(a)}{n!}(z-a)^n + \dots \text{to } \infty$$

Which is convergent at every point inside C

7. Find the Taylor series of the function $f(z)=\sin z$ about $z=\pi/4$ (AU-2013)

$$f(z) = \sin z$$

$$f'(z) = \cos z$$

$$f''(z) = -\sin z$$

$$f'''(z) = -\cos z$$

$$\text{Here } a = \frac{\pi}{4}, f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$f'\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, f''\left(\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$f'''\left(\frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$\text{Taylor's series is } f(z) = f\left(\frac{\pi}{4}\right) + \frac{z - \frac{\pi}{4}}{1!} f'\left(\frac{\pi}{4}\right) + \frac{\left(z - \frac{\pi}{4}\right)^2}{2!} f''\left(\frac{\pi}{4}\right) + \dots$$

8. Find the Laurent's series for the function $f(z) = z^2 e^{1/z}$ about $z=0$ (AU-2013)

$$z^2 e^{\frac{1}{z}} = z^2 \left[1 + \frac{1}{z} + \frac{1}{2!} \frac{1}{z^2} + \dots \right]$$

$$= z^2 + z + \frac{1}{2} + \dots$$

9. Define singular point. (AU-2012)

A point $z=z_0$ at which a function $f(z)$ fails to be analytic is called a singular point or singularity of $f(z)$.

10. Identify the types of singularities of the following function $f(z) = e^{\frac{1}{z-1}}$ (AU-2009)

Here $z=1$ is a singular point

At $z=1$, we get $f(z) = e^{\frac{1}{0}} = \infty$ which is not defined.

Also $z=1$ is not a pole or removable singularity

$z=1$ is an essential singularity.

11. Discuss the nature of the singularities of the function $f(z) = \frac{\sin z}{z}$ (AU-2012)-2

Poles of $f(z)$ are obtained by equating the denominator to zero

$$\text{i.e. } f(z) = \frac{\sin z}{z}$$

$z=0$ is a pole of order 1

$\sin z = 0$

$z = n\pi$ where $n = 0, \pm 1, \pm 2, \dots$

12. Identify the type of singularity of function $\sin(1/1-z)$ (AU-2015)

$$\sin(1/1-z) = 1/1-z - 1/3!(1/1-z)^3 + 1/5!(1/1-z)^5$$

The RHS is the Laurent series with infinite number of terms about the singular part $z=1, z=1$ is an essential singularity of $f(z)$.

13. Find the nature of the singularity $z=0$ of the function $f(z) = \frac{1 - \cos z}{z^2}$ (AU-2011)

Poles of $f(z)$ are obtained by equating the denominator to zero

$$\text{i.e. } f(z) = \frac{1 - \cos z}{z^2}$$

$z^2=0$ is a pole of order 2

14. State Cauchy's residue theorem.

(AU-2014)

If $f(z)$ be an analytic at all points inside and on a simple closed curve C , except for a finite number of isolated singularities $z_1, z_2, z_3, \dots, z_n$ inside C then

$$\int_C f(z) dz = 2\pi i [\text{sum of the residues of } f(z) \text{ at } z_1, z_2, z_3, \dots, z_n] = 2\pi i \sum_{i=1}^n R_i$$

where R_i is the residue of $f(z)$ at $z=z_i$

15. If $f(z) = \frac{-1}{z-1} - 2\left[1 + (z-1) + (z-1)^2 + \dots\right]$, find the residue of $f(z)$ at $z=1$

(AU-2012)

Residue of $f(z)$ at $z=1$ is -1 (the coefficient of $\frac{1}{z-1}$)

16. Find the residue of $\frac{1-e^{2z}}{z^4}$ at $z=0$

(AU-2013)

Given $f(z) = \frac{1-e^{2z}}{z^4}$

Here $z=0$ is a pole of order 4

$$\begin{aligned} \text{Res}(z=0) &= \frac{1}{3!} \lim_{z \rightarrow 0} \frac{d^3}{dz^3} \left[(z-0)^4 \frac{1-e^{2z}}{z^4} \right] \\ &= \frac{1}{6} \lim_{z \rightarrow 0} \frac{d^3}{dz^3} [1-e^{2z}] = -\frac{4}{3} \end{aligned}$$

17. Find the residue of the function $f(z) = \frac{4}{z^3(z-2)}$ at a simple pole

(AU-2012)

$$f(z) = \frac{4}{z^3(z-2)} = \frac{4}{(z-0)^3(z-2)}$$

Here $z=0$ is a pole of order 3 and $z=2$ is a pole of order 1

$$\begin{aligned} \text{Res}(z=0) &= \frac{1}{2!} \lim_{z \rightarrow 0} \frac{d^2}{dz^2} \left[(z-0)^3 \frac{4}{(z-0)^3(z-2)} \right] \\ &= \frac{1}{2!} \lim_{z \rightarrow 0} \frac{d^2}{dz^2} \left[\frac{4}{(z-2)} \right] = \frac{1}{2} \lim_{z \rightarrow 0} \left[\frac{8}{(z-2)^3} \right] = \frac{1}{2} \end{aligned}$$

$$\text{Res}(z=2) = \lim_{z \rightarrow 2} \left[(z-2) \frac{4}{(z-0)^3(z-2)} \right] = \frac{1}{8}$$

18. Find the residue of $f(z) = \frac{z+1}{(z-1)(z-2)}$ at $z=2$

(AU-2012)

$$\text{Res}(z=2) = \lim_{z \rightarrow 2} \left[(z-2) \frac{z+1}{(z-1)(z-2)} \right] = 3$$

19. Find the residue of $\cot z$ at the pole $z=0$.

(AU-2010)

$f(z) = \cot z = \frac{\cos z}{\sin z}$ Poles of $f(z)$ are $\sin z = 0 = \sin n\pi$
 $z = n\pi$, where $n = 0, \pm 1, \pm 2, \dots$

$$[\text{Res}f(z)]_{z=n\pi} = \lim_{z \rightarrow n\pi} (z-n\pi) \frac{\cos z}{\sin z} = \lim_{z \rightarrow n\pi} \frac{-(z-n\pi) \sin z + \cos z(1)}{\cos z} \text{ (by L' Hospital rule)}$$

$$[\text{Res}f(z)]_{z=n\pi} = 1$$

20. Determine the residue of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ at $z=1$ (AU-2012)

Given $f(z) = \frac{z^2}{(z-1)^2(z+2)}$

Here $z=1$ is a pole of order 2

$$\operatorname{Res}[z = z_0] = \lim_{z \rightarrow z_0} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} [z - z_0]^m f(z)$$

$$\begin{aligned} \operatorname{Res}[z = 1] &= \lim_{z \rightarrow 1} \frac{d}{dz} \left[(z-1)^2 \frac{z^2}{(z-1)^2(z+2)} \right] = \lim_{z \rightarrow 1} \frac{d}{dz} \left(\frac{z^2}{z+2} \right) \\ &= f'(z) = \frac{5}{9} \end{aligned}$$

21. Find the residue of $f(z) = \frac{50z}{(z+4)(z-1)^2}$ at $z = 1$ (AU-2009)

$z = 1$ is a pole of order 2

$$\begin{aligned} \operatorname{Res}[f(z)]_{z=1} &= \lim_{z \rightarrow 1} \frac{d}{dz} \left[(z-1)^2 \frac{50z}{(z+4)(z-1)^2} \right] \\ &= \lim_{z \rightarrow 1} \left[\frac{(z+4)50 - 50z}{(z+4)^2} \right] = \frac{250 - 50}{25} = 8 \end{aligned}$$

22. Evaluate $\int_c \frac{e^z}{z-1} dz$ if c is $|z|=2$ (AU-2010)

$z=1$ is a pole of order 1 which lies inside $|z|=2$

$$\begin{aligned} \int_c \frac{e^z}{z-1} dz &= 2\pi i f(1) \\ &= 2\pi i e \end{aligned}$$

PART-B

1. a. Evaluate $\int_c \frac{z}{(z-1)(z-2)^2} dz$ here C is $|z-2| = \frac{1}{2}$ by using Cauchy's integral formula.

(AU-2012)(8)

b. Evaluate $\int \frac{7z-1}{z^2-3z-4} dz$ over the curve $C: x^2+4y^2=4$ using Cauchy's integral formula.

(AU-2013)(8)

2.a. Evaluate $\int_c \frac{z+1}{(z^2+2z+4)^2} dz$ where c is the circle $|z+1+i|=2$ by Cauchy's integral formula.

(AU-2013)(8)

b. Evaluate $\int_c \frac{z+4}{z^2+2z+5} dz$ where C is the circle $|z+1+i|=2$ using Cauchy's integral formula.

(AU-2013)(8)

3.a. Using Cauchy's integral formula, evaluate $\int_c \frac{e^z}{(z+1)^2(z+2)} dz$ where C is $|z|=3$. (8)

b. If $f(a) = \int_c \frac{13z^2+27z+15}{z-a} dz$ where c is the circle $|z|=2$ then find

$$f(3), f'(1-i), f''(1-i) \text{ and } f(1-i) \quad \text{(AU-2014)(8)}$$

4.a. Evaluate $\int_C \frac{z^3}{(2z+i)^3} dz$ where C is the unit circle $|z|=1$ (8)

b. Obtain Taylor's series for $f(z) = \frac{2z^3}{z(z+1)^3}$ about $z=i$ (AU-2013)(8) 5.a. Evaluate

$f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series valid for the regions $|z|>3$ and $1<|z|<3$ (AU-2013)(8)

b. Find the Laurent's series expansion of $f(z) = \frac{7z-2}{(z-2)(z+1)}$ valid in the region $|z+1|<1$ and $|z+1|>3$ (8)

6.a. Expand the function $f(z) = \frac{z^2-1}{z^2+5z+6}$ in Laurent's series $|z|>3$ (AU-2013)(8)

b. Obtain the Laurent's series expansion of $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in $2<|z|<3$ (AU-2015)(8)

7.a. Expand $f(z) = \frac{1}{z^2-4z+3}$ as the Laurent's series expansion of $1<|z|<3$ (AU-2014)(8)

b. Obtain the Laurent's series expansion of $f(z) = \frac{1}{z-z^2}$ in the region $1<|z+1|<2$ and $|z+1|>2$. (AU-2014)(8)

8.a. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is $|z|=3$ Using Cauchy's Residue theorem (AU-2013)(8)

b. Using Cauchy's residue theorem evaluate $\int_C \frac{z-1}{(z-1)^2(z-2)} dz$ where C is $|z-i|=2$ (AU-2014)(8)

9.a. Evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$ where C is $|z|=3$ (AU-2015)(8)

b. Evaluate $\int_0^{2\pi} \frac{dx}{(x^2+a^2)^2}$, $a>0$ using contour integration. (AU-2015)(8)

10.a. Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos \theta} d\theta$ using contour integration (AU-2013)(8)

b. Using contour integration on unit circle, evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\cos \theta}$ (AU-2014)(8)

11.a. Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\sin \theta}$ (AU-2014)(8)

b. Using contour integration, evaluate the integral $\int_0^{2\pi} \frac{\cos 2\theta}{1-2a\cos \theta+a^2} d\theta$ (AU-2013)(8)

12.a. Evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$, $a>0, b>0$ (AU-2013)(8)

b. Evaluate using contour integration $\int_0^{\infty} \frac{dx}{(1+x^2)^2}$ (AU-2014)(8)-2

13.a. Using contour integration prove that $\int_0^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)} = \frac{\pi}{6}$ (AU-2013)(8)

b. Using contour integration on unit circle, evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+9)}$ (AU-2014)(8)

14.a. Evaluate $\int_0^{\infty} \frac{\cos mx}{x^2+a^2} dx$, using contour integration. (AU-2012)(8)

b. Show that $\int_{-\infty}^{\infty} \frac{x^2-x+2dx}{(x^4+10x^2+9)} dx = \frac{5\pi}{2}$ (AU-2013)(8) 15.a.S.T.

$$\int_0^{\infty} \frac{dx}{(1+x^4)} = \frac{\pi}{2\sqrt{2}} \quad (8)$$

b. Evaluate $\int_0^{2\pi} \frac{d\theta}{1-2x\sin\theta+x^2}$ ($0 < x < 1$), using contour integration. (8)

PHYSICS FOR CIVIL ENGINEERING

Unit-I

Thermal performance of buildings

1. define fenestration?

Fenestration is defined as any opening or arrangements of opening , in a building(normally filled with glazing)

2. what is total heat transfer through fenestration?

The heat gain through fenestration consists of two main components:

- Q_{thermal} = heat transfer between indoor and outdoor air.

This is positive or negative depending on temperature.

- Q_{solar} = heat transfer from solar radiation.

This always a positive number.

$$Q_{\text{total}} = Q_{\text{therma}} + Q_{\text{solar}}$$

3. mention few methods of thermal insulation.

Use of materials with low conductivity

Thickness of walls and roofs
Provision of air space
Heat insulation by orientation
Thermal insulation by shading

4. What is thermal comfort?

Thermal comfort is the condition of mind that expresses satisfaction with the thermal environment and is assessed by subjective evaluation. It is the occupants of buildings or other enclosures is one of the important goals of HVAC (heating, ventilation, and air conditioning) design engineers.

5. what is need for shading devices ?

Need of Sun Shading Devices To reduce the heat gains during summer, reduce the cooling loads and therefore minimize energy costs. Sun Shading Devices improve internal environment in order to provide greater comfort for occupants.

6. what is central heating

A central heating system provides warmth to the whole interior of a building (or portion of a building) from one point to multiple rooms.

7. what is ventilation?

Ventilation is the process by which ‘clean’ air (normally outdoor air) is intentionally provided to a space and stale air is removed. This may be accomplished by either natural or mechanical means.

8. what are methods of artificial ventilation?

Exhaust system
Supply system
Combination of exhaust and supply systems

9. what is principle of air conditioning?

Air conditioner continuously draws an air from an indoor space to be cooled and cools it by the refrigeration principles and discharges it back into the same indoor space that needs to be cooled.

10. what is window air conditioner?

Window air- conditioner is also known as room air-conditioner

- It is designed to condition the air in a single room or a large scale.
- It is called a window air-conditioner because it is usually installed in a window.

11. what is packaged air conditioner ?

Packaged air conditioner is self –contained unit primarily for floor mounting, designed to provide conditioned air to the space to be conditioned.

12. what is chilled water plant?

Chilled water is extensively used as a secondary refrigerant in large commercial, institutional and industrial premises to make cooling available over a large area.

13. what is cooling load?

It is defined as the total heat required to be removed from the space in order to bring it to the desired temperature by air conditioning and refrigerating equipment

14. what are the common causes of AC fire?

- Failing to keep the not cleaned properly
- Storing flammable materials near the A.C systems
- Faulty parts and equipments

15. what are the components of central heating system?

A gas supply lines, oil tanks and supply lines or distinct heating supply lines.

A boiler which heats water in the system.

Pump to circulate the water in the closed system

Radiators which are wall-mounted panels through which the heated water pass in order to release heat into rooms

UNIT – II

ACOUSTICS

1. Enumerate the ways in which sound is classified

Sound is classified on the basis of frequency

i) Infrasonics (< 20Hz)

ii) Audible sound(20Hz to 20KHz)

iii) Ultrasonics (> 20Hz)

2. Define intensity of sound. What is its unit?

It is the amount of sound energy flowing per second per unit area held normally at the point to the direction of propagation. Unit: Wm^{-2} .

3. Why are sound waves called mechanical waves?

Since sound waves do some mechanical work while making disturbance in medium, hence sound waves are called mechanical wave.

4. Suppose you and your friend are on the moon. Will you be able to hear any sound produced by your friend?

Since sound waves require medium to propagate and there is no medium present on the moon. So, I will not be able to hear the sound of my friend on the moon.

5. Which wave property determines (a) loudness, (b) pitch?

(a) Amplitude of sound waves determines loudness. Louder sound has greater amplitude and vice versa.

(b) Frequency of the sound waves determines pitch of the sound.

6. Guess which sound has a higher pitch: guitar or car horn?

Sound of the car horn has higher pitch.

7. What are wavelength, frequency, time period and amplitude of a sound wave?

Wavelength: Wavelength is the distance between two consecutive compressions or rarefaction of wave.

Frequency: The number of sound wave produced in one second is called frequency.

Time period: Time period is the time taken to produce one wave of sound.

Amplitude: Amplitude is the maximum displacement along the mean position of the particles of medium.

8. Define Zero electric Resistivity.

zero resistance or superconductivity is the phenomena of making a substance, a super conductor or free of any resistance offered to current when it flows through the object. This phenomena generally occurs at very low temperatures close to zero kelvin! For any object zero kelvin is not possible as it would have zero energy but finite(positive) mass, which is practically and also theoretically impossible.

9. How are the wavelength and frequency of a sound wave related to its speed?

The relation between frequency and wavelength of sound wave is given as follows:

Velocity (v)=Wavelength(λ)X Frequency (ν), $v = \lambda \times \nu$

This means the speed is equal to the product of wavelength and frequency of the sound wave. This equation is also called the 'wave equation' and applicable to all types of wave.

10. Calculate the wavelength of a sound wave whose frequency is 220 Hz and speed is 440 m/s in a given medium.

Given,

$$\text{Frequency } (\nu) = 220 \text{ Hz}$$

$$\text{Velocity } (v) = 440 \text{ m/s}$$

$$\text{Wavelength } (\lambda) = ?$$

$$v = \lambda \times \nu$$

$$\text{Or, } 440 \text{ m/s} = \lambda \times 220 \text{ Hz}$$

$$\text{Or, } \lambda = \frac{440 \text{ ms}^{-1}}{220 \text{ Hz}} = 2 \text{ m}$$

Thus, wavelength = 2m

11. A person is listening to a tone of 500 Hz sitting at a distance of 450 m from the source of the sound. What is the time interval between successive compressions from the source?

Since, the time interval between successive compressions is called time period or time interval.

Here given,

$$\text{Frequency } (\nu) = 500 \text{ Hz}$$

$$T \text{ (Time period)} = ?$$

We know that;

$$\text{Frequency} = \frac{1}{T}$$

$$\text{Or, } T = \frac{1}{\text{Frequency}}$$

$$\text{Or } T = \frac{1}{500 \text{ Hz}} = 0.002 \text{ s}$$

Thus, time interval between two consecutive compression of the given wave = 0.2 s

12. Distinguish between loudness and intensity of sound.

Loudness of sound is determined of amplitude and intensity of the sound wave is determined by frequency of sound waves.

13. In which of the three media; air, water or iron does the sound travel the fastest at a particular temperature?

At particular temperature sound travels fastest in iron.

14. An echo returned in 3 s. What is the distance of the reflecting surface from the source, given that the speed of sound is 342 m/s?

To return an echo sound has to cover distance of two way.

Here, given,

$$\text{Speed of sound} = 342 \text{ m/s}$$

$$\text{Time} = 3 \text{ s}$$

Thus,

$$\text{Distance} = \text{speed} \times \text{time}$$

$$\Rightarrow \text{Distance} = 342 \text{ m/s} \times 3 \text{ s} = 1026 \text{ m}$$

Thus, the distance between the source and reflecting surface = $1026 \div 2 = 513 \text{ m}$

15. Why are the ceilings of concert halls curved?

Since, concert halls are big, so audience at the back rows of the hall may not hear clear sound of speaker. To overcome this problem, the ceiling of the concert halls is made concave. Concave ceiling helps the sound wave to reflect and send to farther distance which makes the concert hall enable to send clear sound to the audience even sitting in back rows of hall.

16. What is the audible range of the average human ear?

20 Hz to 20000 Hz

17. What is the range of frequencies associated with (a) Infrasound (b) Ultrasound

(a) Infrasound: Less than 20 Hz

(b) Ultrasound: More than 20000 Hz

18. A submarine emits a SONAR pulse, which returns from an underwater cliff in 1.02 s. If the speed of sound in salt water is 1531 m/s, how far away is the cliff ?

To return the SONAR pulse back, its wave has to travel two way.

Here, given,

$$\text{Velocity (v) of sound wave} = 1531 \text{ m/s}$$

$$\text{Time (T)} = 1.02 \text{ s}$$

Thus, Distance = speed \times time

$$\text{Distance} = 1531 \text{ ms}^{-1} \times 1.02 \text{ s} = 1561.62 \text{ m}$$

So, the distance between the source and reflecting surface = $1561.62 \div 2 = 780.81 \text{ m}$

19. Cite an experiment to show that sound needs a material medium for its propagation.

Activity:

- Take a glass bell jar, connect it with vacuum pump and suspend an electric bell in it.
- Connect electric bell with a battery.
- Switch on the electric bell and hear the sound of bell.
- Now remove the air completely from the bell jar using vacuum pump and observe the sound of electric bell.

20. Why is sound wave called a longitudinal wave?

Since sound wave creates oscillation in the particles of the medium parallel to the disturbance in the direction of propagation, thus sound waves are called longitudinal wave. This would be more clear by taking the definition of longitudinal wave into account.

Longitudinal wave: When oscillation is created parallel to the disturbance of the particles of medium in the direction of propagation.

21. Which characteristic of the sound helps you to identify your friend by his voice while sitting with others in a dark room?

Timbre and pitch are the characteristics of sound which help to identify the sound of different voice. Thus, because of difference in timbre and pitch of the sound wave I or any other can identify the voice of his friend sitting with others even in dark room.

22. Flash and thunder are produced simultaneously. But thunder is heard a few seconds after the flash is seen, why?

This happens because of the difference in the velocity of light and sound waves. Light travels with much faster velocity than sound. That's why thunder is heard a few seconds after the flash of thunder is seen instead of both are produced simultaneously.

23. A person has a hearing range from 20 Hz to 20 kHz. What are the typical wavelengths of sound waves in air corresponding to these two frequencies? Take the speed of sound in air as 344 m/s.

Given, velocity of sound = 344 m/s

We know that

Velocity = wavelength X frequency

$$\Rightarrow \text{Wavelength} = \frac{\text{Velocity}}{\text{Frequency}}$$

Thus, in the case of sound of frequency of 20 Hz

$$\text{Wavelength} = \frac{344 \text{ m/s}}{20 \text{ Hz}} = 17.2 \text{ m}$$

In the case of sound of frequency of 20 kHz = 20000 Hz

$$\text{Wavelength} = \frac{344 \text{ m/s}}{20000 \text{ Hz}} = 0.00172 \text{ m}$$

24. Two children are at opposite ends of an aluminium rod. One strikes the end of the rod with a stone. Find the ratio of times taken by the sound wave in air and in aluminium to reach the second child.

We know that,

The speed of sound in air = 344 m per second

The speed of sound in aluminium = 5100 m per second

Hence, the ratio of time taken by the sound to travel through air and through aluminium

$$\begin{aligned} &= \frac{\text{Velocity of sound in aluminium}}{\text{Velocity of sound in air}} \\ &= \frac{5100 \text{ m per second}}{344 \text{ m per second}} = 150 : 1 \end{aligned}$$

25. The frequency of a source of sound is 100 Hz. How many times does it vibrate in a minute?

Given, frequency = 100 Hz

This means the source of sound vibrates 100 times in one second.

Therefore, number of vibrations in 1 minute, i.e. in 60 seconds = 100 x 60 = 6000 times.

26. Does sound follow the same laws of reflection as light does? Explain.

Yes, the sound wave follows the same laws of reflection as the light does. The laws of reflection of sound are as follows:

- The incident sound wave, the reflected sound wave and the normal at the point of incident, all lie in the same plane.
- The angle of incidence of sound wave and angle of reflection of sound wave to the normal are equal.

When sound waves reflected from a surface, the angle of incidence is equal to the angle of reflection to the normal and the incident wave, normal and reflected wave are in the same plane. This can be proved by experiment. Thus, sound wave obeys the laws of reflection.

27. When a sound is reflected from a distant object, an echo is produced. Let the distance between the reflecting surface and the source of sound production remains the same. Do you hear echo sound on a hotter day?

To hear the sound of echo depends upon the distance from source of sound and reflecting surface. The distance between both must be equal to or more than 17.2 meter. If the given distance is more than 17.2 meter then one can hear the echo sound on a hotter day also.

Although, in hotter day the velocity of sound increases, thus it is necessary to hear the sound of echo the distance should be more than 17.2 meter. If the given distance is equal to 17.2, then to hear the sound in hotter day would not be possible.

28. Give two practical applications of reflection of sound waves.

Bulb horn and Stethoscope are examples of practical applications of reflection of sound waves. In bulb horn sound is amplified and sent to the desired direction because of reflection. In stethoscope also sound is sent to the desired direction because of its reflection characteristic.

29. A stone is dropped from the top of a tower 500 m high into a pond of water at the base of the tower. When is the splash heard at the top? Given, $g = 10 \text{ m}^{-2}$ and speed of sound = 340 m/s.

Given,

Height of tower = 500 m

$g = 10 \text{ m/s}^2$

Velocity of sound = 340 m/s

Thus, to calculate the time of splash sound, first of all time taken to reach the stone in the water is to be calculated.

$$s = ut + \frac{1}{2}at^2$$

$$\text{Or, } s = ut + \frac{1}{2}gt^2$$

Here,

$$s = 500 \text{ m and } g = 10 \text{ m/s}^2$$

$$\Rightarrow 500 \text{ m} = 0 \times t + \frac{1}{2} \times 10 \text{ ms}^{-2} \times t^2$$

$$\Rightarrow 500 \text{ m} = 5 \text{ ms}^{-2} \times t^2$$

$$\Rightarrow t^2 = \frac{500 \text{ m}}{5 \text{ m s}^{-2}} = 100 \text{ s}^2$$

$$\Rightarrow t = 10 \text{ s}$$

Now, we know that the distance = speed X time

$$\Rightarrow \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\Rightarrow \text{Time} = \frac{500 \text{ m}}{340 \text{ ms}^{-1}} = 1.47 \text{ s}$$

So, total time to hear the sound of splash = 10 s + 1.47 s = 11.47 sec

30. A sound wave travels at a speed of 339 m/s. If its wavelength is 1.5 cm, what is the frequency of the wave?

Given,

Velocity (v) of sound = 339 m/s

Wavelength (λ) = 1.5 cm = 0.015 m

Frequency (ν) = ?

We know that, speed = wavelength X frequency

$\Rightarrow 339 \text{ m/s} = 0.015 \text{ m} \times \text{frequency}$

$$\Rightarrow \text{Frequency} = \frac{339 \text{ m/s}}{0.015 \text{ m}} = 22600 \text{ Hz}$$

Thus, frequency = 22600 Hz

UNIT III LIGHTING MATERIALS

1) Define Radiance.

Radiance is defined as intensity of optical radiation

Emitted or reflected from a certain location on an emitting or reflecting surface in a particular direction $d\Phi = L_c \cos \theta dA d\Omega$.

2) Define Irradiance.

Irradiance is defined as the amount of radiant power Impinging upon a surface per unit area.

The radiant power

$$d\Phi = E_e dA.$$

3) Define radiant exitance

Radiant exitance M_e quantifies as the amount of radiant power per unit area on a surface. The radiant power $d\Phi = M_e dA$.

4) What is spectral radiant power?

It is defined as a source's radiant power per wavelength interval as a function of wavelength.

The radiant power $d\Phi$ Emitted in the wavelength interval between λ and $\lambda + d\lambda$ is given by $d\Phi_e = \Phi_\lambda(\lambda)d\lambda$

5) What is photometry ?

The branch of optics which deals with the measurements of the intensity of light emitted by a source, its illuminating property or intensity of illumination of a surface is called As photometry.

6) Define Lumen

Lumen is defined as the luminous flux emitted from the standard candle.

7) Define Luminous flux:

The light energy emitted per second from a light source

is called as luminous flux It is denoted by Φ . Unit of Luminous flux is Lumen (lm).

8) Define Luminous Intensity:

Luminous Intensity is the illuminating power of a source in any direction is defined as the luminous flux emitted per unit solid angle In that direction.

9) Write the formula for Luminous Intensity

$$I = \text{Lumen/Steradian (or) Candela.}$$

10) Define candela :

It is defined as the unit of luminous intensity of a source. A light has a luminous intensity of 1 candela if it emits 1 lumen per steradian.

11) State Cosines Law

It states that the intensity of illumination is

(i) Directly proportional to the cosine of the angle of incidence of light radiation.

(ii) Inversely proportional to the square of distance between the surface and source.

UNIT IV
NEW ENGINEERING MATERIALS

1. What is a composite?

A composite is structural materials that consist of two or more combined constituents that are combined at a macroscopic level and are not soluble in each other. One constituent is called the reinforcing phase and the one in which it is embedded is called the matrix. The reinforcing phase material may be in the form of fibres, particles, or flakes. The matrix phase materials are generally continuous. Examples of composite systems include concrete reinforced with steel and epoxy reinforced with graphite fibres, etc.

2. What are advanced composites?

Advanced composite materials are manmade composite materials. They include as resin, metal or ceramic matrix composite systems reinforced with high strength, high modulus continuous or discontinuous forms (such as continuous fibres, chopped fibres, whiskers or particles). The resultant material exhibits entirely distinct, but superior properties which are not exhibited by the constituting elements. Advanced composites are composite materials that are traditionally used in the aerospace industries. These composites have high performance reinforcements of a thin diameter in a matrix material such as epoxy and aluminum.

3. What are natural composites? Give examples.

The composite material that exists in nature is regarded as natural composites. Examples of such composites include, wood (composed of cellulose and lignin), human or animal body (composed of bones and tissues) or even rocks and materials. Bones themselves are composites made of two components, the organic and inorganic. The organic component consists largely of carbohydrates, fats and proteins imparting pliability to the bones. The organic component is calcium phosphate which provides rigidity and strength to the bones. Sea shells and elephant tusk are also natural layered composites.

4. What are the common types of composites?

The most typical types of composites consist of an additive or reinforcement such as fibres or particles embedded in a supporting material called matrix. Usually these are structural materials but they can also be special materials such as electrical conductors. Some composites have no matrix and are composed of one or more constituent forms consisting of two or more different materials. Laminates, for example, are composed entirely of layers which taken together, give the composite its form. Plywood with multiple plies with different orientation of fibres is a typical laminate composite. Sandwiches which are composed of a thick and light core material bonded on either side by a thin strong facing are also regarded as composites. Many felts and fabrics have no body matrix but consist entirely of fibres of several compositions with or without bonding force.

5. What is a reinforced composite?

A reinforced composite is one in which the primary purpose of the insert is to improve the mechanical properties of the composite. The insert is called reinforcement. The composite is formed is called reinforced composite

6. What is Filled composite? Give some examples of fillers commonly used in composites.

A filled composite consists of an insert whose main function is to alter the physical property other than the mechanical properties. Such an insert is called filler. Addition of filler also reduces the cost of composite. Commonly used fillers include carbon black, calcium carbonate, mica, talc, barium carbonate, calcium sulphate, china clay and glass beads (also called spheres of glass).

7. What are the general requirements of all composite materials?

All the composite materials must meet the following requirement. The reinforcing phase must be distributed uniformly throughout the matrix and the reinforced particles and fibres must not be in direct contact with one another. The reinforcing phase must not react with the matrix at the service temperature as this will affect the interfacial bond with the matrix. The reinforcing phase must not lose its strength and should be well bonded to the matrix. Matrix must have lower modulus of elasticity as compared to the reinforcing phase.

8. List the various functions that a matrix phase performs in a composite material.

A matrix phase performs the following main functions: It binds the reinforcement in place in the structure and protects it from mechanical and chemical damage that might occur by abrasion of their surface or by chemical attack or some extraneous source. It separates the individual fibres and particles and deflects brittle crack that otherwise can pass across the entire cross section of the composite. The matrix takes the load and transfers it to the

reinforcement in case of fibre reinforced or particle reinforced composites. The matrix also gives shape to the composite. In some instances the matrix controls the electrical and chemical properties of the composite.

9. What is the role of a coupling agent present in the interface between reinforcement and matrix?

A matrix-reinforcement interface plays an important role in a composite as the load is transferred from matrix to the fibres through this interface. The eventual properties of a composite are greatly dependent on the nature of the interface. The desired characteristics of the interface are controlled by a third material called the coupling agent or compatibilizer. The coupling agent can overcome the weak interaction between the matrix and the reinforcement. The matrix and the reinforcement differ in their chemical nature and surface characteristics but the addition of coupling agent results in their association, leading to improve strength of the resultant composite.

10. Differentiate between dispersion strengthened and particle strengthened composites.

Dispersion strengthened composites contain particles up to about 15% by volume having size in the range $0.01\mu\text{m}$ to $0.1\mu\text{m}$ whereas the matrix of particle reinforced composites contains particles in amounts from 15% to 40% by volume with size greater than $1.0\mu\text{m}$. In special cases, particle content may be as high as 94%. In particulate composites both the matrix and particles share the load equally whereas the matrix carries the major load in dispersion strengthened composites.

11. What is the basic principle used in the fibre reinforced composites?

The basic principle used in fibre reinforced composites is that the materials are generally stronger in fibre form than in bulk form. This is due to the alignment of molecules along the fibre axis. This preferred alignment makes the strength and modulus of both natural and synthetic fibres superior to those of the same material in the randomly oriented bulk form. That is why major part of the load is carried by the fibres and not by the bulk matrix. As fibres are to carry the major portion of the applied load they should be much stronger and stiffer than the matrix. This property is also desirable for preventing the fibres from buckling under compressive stresses.

12. What are 'whiskers'?

Name some whiskers that are currently available. Whiskers are very fine single crystal materials with diameters of the order of submicron. Their length may range from a fraction of a millimeter to several centimeters. The aspect ratio (i.e. the length to diameter ratio) of whiskers generally ranges from 50 to 10000. Due to very fine diameter they contain very few or no dislocations and their strength approaches that of an ideal or perfect crystal (i.e. the theoretically achievable limit). Their resistance against temperature and strain to failure are very much higher than high performance fibres. The elastic strain to fracture may be as high as 5%. However, they suffer from the problem of brittleness and are difficult to process. Metals, oxides, carbides, halides and organic compounds have been grown in whisker form. SiC, Si₃N₄, carbon, alumina and K₂O.6TiO₂ whiskers are currently available. Among these, SiC whiskers seem to be the best suited for metal-matrix composite reinforcement.

13. Define Flake Composites.

Flakes are often used in place of fibers as they can be densely packed. Metal flakes that are in close contact with each other in polymer matrices can conduct electricity or heat, while mica flakes and glass can resist both. Flakes are not expensive to produce and usually cost less than fibers. Flake composites have a higher theoretical modulus of elasticity than fiber reinforced composites. They are relatively cheaper to produce and handled in small quantities.

14. Name the various grades of glass fibres used as reinforcement in glass fibre reinforced plastic (GRP).

There are three grades of glass fibres available for reinforcement in glass fibre reinforced plastics. They are, E-glass (or electrical glass), C-glass and S-glass.

15. List the types of fibres used as reinforcements in composites.

Organic fibres such as nylon and aramid; inorganic fibres of glass, graphite and ceramics; and stainless steel fibres and wires of tungsten are some typical fibre materials used as reinforcement in composites.

16. Give some of the characteristics of C-fibres.

C-glass (or chemical glass), is similar to E-glass in composition and provides excellent resistance to chemical, especially acids. When combined with E-glass reinforcement, this glass is very useful for surfacing mats for many applications that require corrosion resistance.

17. What are the disadvantages of glass fibres?

The disadvantages of glass fibres are low modulus, self-abrasiveness which frequently decreases the potential strength, relatively low fatigue resistance and poor adhesion to matrix resins. This means that they do not impregnate the matrix well and uniformly. Therefore, these fibres are never used for high performance composites.

Through this adhesively can be improved by sizing on the surface, this may provide a path for absorption of moisture and create undesirable internal stresses.

18. Define Solid Microspheres.

Solid Microspheres have relatively low density, and therefore, influence the commercial value and weight of the finished product. Studies have indicated that their inherent strength is carried over to the finished molded part of which they form a constituent. Solid glass Microspheres, manufactured from glass are most suitable for plastics. Solid glass Microspheres are coated with a binding agent which bonds itself as well as the sphere's surface to the resin. This increases the bonding strength and basically removes absorption of liquids into the separations around the spheres.

19. What are the disadvantages of carbon fibres?

In spite of highly desirable characteristics, carbon fibres have certain disadvantages. They have low impact toughness and high electrical conductivity. The latter property may cause „shortening“ in unprotected electrical machinery. Another disadvantage of carbon fibres is their high cost. Due to high cost they are sometimes used in hybrid composites, along with relatively cheaper fibres such as glass.

20. Define Microspheres.

Microspheres are considered to be some of the most useful fillers. Their specific gravity, stable particle size, strength and controlled density to modify products without compromising on profitability or physical properties are its most sought after assets. They are of two types of microspheres they are solid microspheres and hollow microspheres.

21. What are Kevlar fibres? What are the applications for which these fibres were developed originally?

Kevlar is the trade name of aramid fibres developed at Du Pont Company. The two varieties of Kevlar that are most common include „Kevlar49“ and „Kevlar29“. These fibres were originally developed to replace steel wire used in radial tyres. Kevlar is about five times stronger than steel on weight to weight basis.

22. Define Filled Composites.

Filled composites result from addition of filler materials to plastic matrices to replace a portion of the matrix, enhance or change the properties of the composites. The fillers also enhance strength and reduce weight. Fillers may be the main ingredient or an additional one in a composite. The filler particles may be irregular structures, or have precise geometrical shapes like polyhedrons, short fibers or spheres.

23. What is a carbon-carbon composite produced?

A carbon-carbon composite material consists of a carbonaceous matrix reinforced with carbon fibres in the form of continuous filament yarns, cloth, chopped fibres or three dimensional woven reinforcement.

24. What are aramid fibres?

Aramid is the generic term for fibres produced from „aromatic polyamides“. Polyamides are long chain polymers containing aromatic rings or so called as benzene rings. These rings recur repeatedly in the structure of aramid fibres.

25. How is carbon fibres produced?

Carbon fibres are produced by pyrolysis of suitable carbonaceous products such as polyacrylonitrile (PAN), pitch and staple rayon fibres called precursors. The process involved three stages, namely, stabilization, carbonization and graphitization.

26. List some typical ceramic matrix composite systems.

Typical ceramic matrix composite systems include carbon-carbon composite, molybdenum disulphide, SiC matrix/ carbon fibre composite, lithium aluminosilicate matrix/ Nicalon (SiC) fibre, and ceramic matrix/ glass fibre composite.

27. What are the advantages of thermo plastic matrices?

Superior toughness. Short moulding cycle. Capability of fusion bonding. Infinite Prepreg shelf life. Easy repairability. Reusability of scrap.

28. List the disadvantages of thermoplastic matrices.

Higher temperature of processing. Relatively low heat resistance. Sensitivity to solvents.

29. What are the most common advanced composites?

The most common advanced composites are polymer matrix composites (PMCs) consisting of a polymer (e.g., epoxy, polyester, urethane) reinforced by thin diameter fibers (e.g., graphite, aramid, boron). The reasons why they are the most common composites include their low cost, high strength, and simple manufacturing principles.

30. What are hollow microspheres?

Hollow microspheres are essentially silicate based, made at controlled specific gravity. They are larger than solid glass spheres used in polymers and commercially supplied in a wider range of particle sizes. Commercially, silicate-based hollow microspheres with different compositions using organic compounds are also available.

31. What are the drawbacks of polymer matrix composites?

The main drawbacks of PMCs include low operating temperatures, high coefficient of thermal and moisture expansion, and low elastic properties in certain directions.

32. What are the principal fibers used in commercial applications?

The principal fibers in commercial use are various types of glass and carbon as well as Kevlar 49. Other fibers, such as boron, silicon carbide, and aluminium oxide, are used in limited quantities.

33. What are the influences of fiber in a composite laminates?

Density. Tensile strength and modulus. Compressive strength and modulus. Fatigue strength as well as fatigue failure mechanisms. Electrical and thermal conductivities.

34. What are the principal advantages of glass fibers?

Low cost. High tensile strength. High chemical resistance. Excellent insulating properties.

35. Define the term 'wettability'.

The adhesion between the reinforcing fiber and the matrix in composite materials plays an important role in the final mechanical properties of the material. It is called as wettability.

36. Define Cermets/Ceramal.

The Cermets is an abbreviation for the "ceramic" and "metal." A Cermets is a composite material composed of ceramic (Cer) and metallic (Met) materials. Cermets are ideally designed to have the optimal properties of both a ceramic, such as high temperature resistance and hardness, and those of a metal, such as the ability to undergo plastic deformation. The metal is used as a binder for an oxide, boride, carbide, or alumina. Generally, the metallic elements used are nickel, molybdenum, and cobalt. Depending on the physical structure of the material, cermets can also be metal matrix composites, but cermets are usually less than 20% metal by volume. It is used in the manufacture of resistors (especially potentiometers), capacitors, and other electronic components which may experience high temperatures.

37. What are fiber metal laminates?

Fiber-reinforced polymer laminas can also be combined with thin aluminium or other metallic sheets to form metal-composite hybrids, commonly known as fiber metal laminates (FML).

38. What are coupling and coating agents?

Coupling agents and coatings are applied on the fibers to improve their wetting with the matrix as well as to promote bonding across the fiber – matrix interface. Both in turn promote a better load transfer between the fibers and the matrix.

39. What are the roles of matrix in FRP?

To keep the fibers in place. To transfer stress between the fibers. To provide a barrier against an adverse environment, such as chemicals and moisture. To protect the surface of the fibers from mechanical degradation.

40. What are the classifications of polymers?

Thermoplastics
Thermosets.

41. What will happen when load applied to FRP at glass transition temperature?

When an external load is applied, it exhibits an instantaneous elastic deformation followed by a slow viscous deformation followed by a slow viscous deformation. With increasing temperature, the polymer changes into a rubber like solid capable of undergoing large, elastic deformations under external loads.

42. What is Interface?

The bonding between the fiber and the resin is called interface. It is a region where coating and matrix diffused into each other's domain and form a flexible, three-dimensional polymer network.

43. Define the terms filament, strand, yarn.

Filament is defined as any fibre whose aspect ratio (length to diameter ratio) is for all practical purposes, infinity. Filament is thus a continuous fibre. The untwisted form of glass fibers is called as strand. A group of 200 or more fibers are called as strand. Strand is a primary bundle of continuous filaments combined in a single compact unit without twist. Yarn is a generic term used for strands of fiber or filaments in a form suitable for weaving.

44. What is roving?

A roving is a group of untwisted parallel strands wound on a cylindrical forming package. Roving's are used in continuous moulding operations, such as filament winding and pultrusion.

45. List various ceramic matrices.

Aluminium oxide (Al_2O_3). Carbon, silicon carbide (SiC). Silicon nitride (Si_3N_4).

46. Do Shape Memory Alloys (SMA) work harden?

When we change the shape of a SMA, its internal crystalline structure rearranges itself to another form. But when we heat up the SMA, it uses the heat energy to rearrange its crystalline structure to its original form, thus regaining its original shape. In the end, you can say that the work done in hardening it will be undone when it returns to its original state.

UNIT V HAZARDS

1. What are the causes of hazards?

Some of the hazards which cause environmental degradation are floods, earth quakes, cyclone, landslides, drought, famines.

2. What the types are hazards?

i) natural hazards ii) Anthropogenic hazards

3. What are the natural hazards?

Air related hazards: Hurricanes, cyclones, storms

Water related hazards: Floods, drought

Earth related hazards: Earthquakes, landslides, volcanoes.

4. What are the anthropogenic hazards?

i) industrial accidents

ii) War, riots, acts of terrorism

iii) Toxic spills

5. What is earthquake?

An earthquake is caused by a portion of the rigid crust of the earth giving way, or getting fractured, some distance below its surfaces.

6. Define four and epicenter earthquake?

The place where the actual fracture occurs is called the focus of the earthquake. It is not a geometrical point, but an extended region. The point nearest to the focus, on the surface of the earth is called the epicenter.

7. What are the causes of earthquake?

i) sudden movement of hot gases and magma

ii) Volcanic activities

iii) Stress caused by water pressure in dams

iv) Tectonic stress generated by movement of tectonic plates.

8. What are types of earthquakes?

i) shallow focus earthquakes

ii) intermediate focus earthquake

iii) deep focus earthquakes

9. Define intensity of earthquakes?

Intensity is a qualitative measure of the strength of an earthquake. It gives a gradation strength of earthquake. Using observed damage to structure and ground and reaction of humans to the earthquake shaking.

10. Define magnitude of earthquake.

The magnitude is a quantitative of absolute measure of the size of an earthquake it can be correlated to the amount of wave energy realised at the source of an earthquake.

11. What are the effects earthquake?

- i) ground shaking
- ii) liquefaction of ground
- iii) ground displacement
- iv) landslides
- v) flood

12. What are the types of seismic waves?

- i) priliminary waves
- ii) surface waves

13. What is primary waves?

These are longitudinal waves in which the particles of earth vibrate about their mean position along the directions of the waves themselves.

PART B QUESTIONS

Unit-I

THERMAL PERFORMANCE OF BUILDINGS

1. Discuss heat transfer through fenestration.
2. Describe methods of thermal insulations.
3. Explain heat gain and loss estimation in the components of buildings.
4. Discuss the factors affecting thermal performance of buildings.
5. Describe thermal measurements in buildings.
6. Explain thermal comfort and indices in buildings.
7. Describe climate and design of solar reclusion.
8. Discuss the different types of shading devices.
9. Explain central heating systems.
10. Discuss the principles of natural ventilation, ventilations measurements and design for natural ventilations.
11. Describe construction and working of window air conditioner.
12. Write a note on (a) packaged air conditioner r (b) chilled water plant.

13. Write a note on (a) Fan coil units (b) water piping.
14. Discuss different air conditioning systems for buildings.
15. What are the common causes of AC fires? What are steps to be taken to prevent the fire.

Unit-II ACOUSTICS

1. State and explain sabine's formula for reverberation time of a hall. Derive sabine's formula for reverberation time.
2. Derive an expression for the reverberation period of an auditorium and explain how this can be used for determining the absorbing power of surface involved.
3. Define absorption coefficient in sound. Describe a method of measuring the absorption coefficient of a material.
4. Discuss the factors reverberation, resonance, echelon effect and focusing that acoustics in a hall. Give remedies.
5. Write in detail about the factors affecting architectural acoustics and their remedies.
6. Discuss the factors reverberation, resonance, echelon effect, focusing and reflection that acoustics in a hall and the remedies for them.
7. What is reverberation time? Using sabine's formula explain the sound absorption coefficient of a material is determined.
8. Derive an expression for growth and decay of sound energy.
9. Derive an expression for growth and decay of sound energy density inside a hall and hence deduce sabine's formula for the reverberation time of a hall.

Unit-III LIGHTING DESIGNS

1. Discuss the different radiometric quantities.
2. Explain the different photometric quantities.
3. Derive cosines law and inverse square law in photometry.
4. Describe relations between radiant and luminous characteristics of radiations.
5. Explain colour- luminous efficiency function.
6. Write a note on (a) visual field glare (b) day light calculations (c) day light factor.
7. Discuss daylight design of windows.
8. Describe principles of artificial lighting and supplementary artificial lighting.

Unit IV NEW ENGINEERING MATERIALS

1. Discuss the classification of composites. Give detailed study of fibers reinforced plastics (FRP) and reinforced metal (FRM).
2. Explain the preparation, types, properties and applications of metallic glasses.
3. Describe the type properties and applications of shape memory alloys.
4. Discuss the classification of ceramics.
5. Explain the following manufacturing methods of ceramics 1. Slip casting 2. Isostatic pressing 3. Gas pressure bonding
6. Explain thermal mechanical, electrical and chemical properties of ceramics materials.

7. Write a note on (a) Ceramic fibers (b) Ferro electric ceramics (c) Ferrromagnetic ceramics (d) High aluminium ceramics.

Unit V
HAZARDS

1. Discuss earthquake ground motion with types , intensity and magnitude.
2. Explain seismic waves and seismology.
3. Describe earthquake site effects.
4. Explain deterministic seismic hazards analysis and probabilistic seismic hazard analysis.
5. Explain cyclone hazards with cause and effect. Write prevention measures of cyclone.
6. Explain food hazards. Mention the effects and methods of food prevention.
7. Explain fire hazards and fire protections.
8. Describe fire proofing of materials.
9. Discuss fire fighting equipments.
10. Write a note on fire safety regulations.
11. Discuss fire prevention and safety measures.
12. Explain why some tropical and subtropical areas experience a greater frequency and risk of tropical cyclones and seismic activity than others.
13. Natural Hazards have a greater impact on LEDC's than MEDC's discuss.

ENVIRONMENTAL SCIENCE AND ENGINEERING

Environmental Science and Engineering

UNIT I – ENVIRONMENT, ECOSYSTEMS AND BIODIVERSITY

1. Define environment.

Environment is defined as the sum of total of all the living and non-living things around us influencing one another.

2. What are all the categories of environment?

The main categories of environment are biotic and abiotic environments. The abiotic environment can further be classified into atmosphere (air), lithosphere (soil), and hydrosphere (water). The biotic environment is called as biosphere.

3. Write the components of environment?

Air (Atmosphere)

Land (Lithosphere)

Water (Hydrosphere)

4. Define ecosystem.

A group of organisms interacting among themselves and with environment is known as ecosystem. Thus an ecosystem is a community of different species interacting with one another and with their nonliving environs exchanging energy and matter.

5. List any four characteristics of ecosystem.

(i). Ecosystem is the major ecological unit.

(ii). It contains both biotic and abiotic components.

(iii). The boundary of the ecosystem is not rigidly defined and it is flexible.

(iv). Through the biotic and abiotic components nutrient cycle and energy flow occur.

6. What are the different types of ecosystem

1. Natural Ecosystem

2. Artificial Ecosystem

3. Incomplete Ecosystem

7. Write about autotrophic organisms.

The producers are plants and bacteria, capable of producing their own food photosynthetically or by chemical synthesis. These organisms are, thus, self-nourishing as they can produce the as they can produce their own food by using the energy from the physical environment surrounding them .them are also know as autotrophic organism or producer.

8. Write about heterotrophic organisms

These organisms depend on other animals or on the food produced by other organisms for their nourishment, they are also called as heterotrophic organisms or consumers.

9. What do you know about food chain and food web?

A food chain is a picture or model that shows the flow of energy from Autotrophs to a series of organisms in an environment. The network like interaction of organisms is called as food web.

10. Define food chain.

The sequence of eating and being eaten in an ecosystem is known as food chain or transfer of food energy from the plant through a series of organisms is referred to as food chain.

11. Name three types of food chains.

- a. Grazing food chain
- b. Detritus or Decomposer food chain
- c. Parasitic food chain

12. Define food web.

The interlocking pattern of various food chain in an ecosystem is known as food web.

13. Define ecological pyramids.

Graphical representation of structure and function of trophic levels of an ecosystem, starting with producers at the bottom and each successive trophic levels forming the apex is known as an ecological pyramids.

14. Define primary succession.

The succession taking place in areas that have not already been occupied by any community is known as primary succession.

15. Define secondary succession.

Development of a new community in an area where the previously existing community was removed and the ecological conditions are favourable is termed as secondary succession.

16. Define biodiversity.

Biodiversity is defined as the variety and variability among all group of living organisms and the ecosystem in which they occur.

17. What are the three types of biodiversity?

1. Genetic Level or Genetic diversity
2. Species Level or Species diversity
3. Ecosystem Level or Ecosystem diversity.

18. Define genetic diversity.

Genetic diversity is the variation of genes within species. Genes are the basic units of all life on earth. They are responsible for both the similarities and the differences between organisms.

19. Define species diversity.

Species diversity is the number of different species of living things available in an area. Species is a group of plants or animals that are similar and able to breed and produce viable offspring under natural conditions. This type of diversity is the most common level of diversity.

20. Define ecosystem diversity.

Ecosystem diversity is the variety of ecosystems in a given place. An ecosystem is a community of organisms and their physical environment interacting together. An ecosystem can cover a large area, such as a whole forest, or a small area, such as a pond.

21. What are the two main functions of biodiversity?

- a. It is the source of species on which the human compete depends for food, fiber, shelter, fuel and medicine.
- b. It depends on the biosphere, which in turn leads to the stability in climate, water, soil, air, and the overall health of biosphere.

22. Define biogeography.

The study of the geographical distribution of biological species relating to the geological, evolutionary, climatological, geographical, biological reasons for the distribution is called biogeography. Define biogeography. The study of the geographical distribution of biological species relating to the geological, evolutionary, climatological, geographical, biological reasons for the distribution is called biogeography.

23. Biodiversity hotspots are areas:

- i. Rich in plant and animal species, particularly many endemic species, and
- ii. Under immediate threat from impacts such as land clearing, development pressures, salinity, weeds and feral animals.

24. List some of the major biodiversity threats. a. Habitat destruction.

b. Extension of agriculture. c. Filling up of wetlands.

d. Conversion of rich bio-diversity site for human settlement and industrial development.

25. What do you know about conservation of biodiversity?

Conservation is defined as the management of human use of the biosphere so that it may yield the greatest sustainable benefit while maintaining its potential to meet the needs and aspirations of future generations".

26. What are the two types of biodiversity conservation?

- a. In-situ conservation
- b. Ex-situ conservation

27. What is endemic species?

The species, which are found only in particular region are known as endemic species

28. Define in-situ conservation.

In-situ conservation involves protection of fauna and flora within its natural habitat, where the species normally occurs is called in-situ conservation.

29. Define ex-situ conservation.

Ex-situ conservation involves protection of fauna and flora outside the natural habitats.

UNIT II – ENVIRONMENTAL POLLUTION

1. Define pollution.

Environmental pollution may be defined as the unfavorable alteration of our surroundings its change the quality of the air, water and land which interferes with the health of humans and other life on earth.

2. Types of pollutants.

Biodegradable pollutants

3. Define air pollution.

Air pollution is defined as the presence of one or more contaminants like dust, smoke, mist and odor in the atmosphere which are injurious to human beings, plants and animals.

4. What are the different sources of air pollution?

The two main sources of air pollution are a. Natural Sources.

Natural sources include dust storms, volcanoes, lightening sea salt, smoke, and forest fires.

b. Man made or anthropogenic sources.

The man made sources are agricultural activities, industrial growth, domestic wastes, automobile exhausts, etc.

5. Define photo chemical smog.

A photochemical reaction is any chemical reaction activated by light that air pollution knows as photochemical smog is a mixture of more than 100 primary and secondary pollutants formed under the influence of sun light. Its formation begins inside automobile engines and the boilers an in coal-burning power and industrial plants.

6. What do you know about particulate?

In general the term „particulate“ refers to all atmospheric substances that are not gases. They can be suspended droplets or solid particles or mixtures of the two. Particulates can be composed of materials ranging in size from 100mm down 0.1 mm and less. The chemical composition of particulate pollutants is very much dependent upon the origin of the particulate.

7. Define suspended particulate matter

Suspended Particulate Matter (SPM) is a complex mixture of small and large particles with size less than 100u varying origin and chemical composition.

8. Differentiate between Mist and Fog.

Mist is made up of liquid droplets generally smaller than 10um which are formed by condensation in the atmosphere or are released from industrial operations.

Fog is similar to mist but the droplet size bigger (> 10u) and water is the liquid. Fog is sufficiently dense to incomprehensible vision.

9. What are effects of air pollution on animals?

Animals take up fluorides of air through plants. Their milk production falls and their teeth and bones are affected. They are also prone to lead poisoning and paralysis.

10. List some of the effects of air pollution on physical properties of atmosphere.

- a. Decrease in the visibility
- b. Reduction of Solar radiation.
- c. Effects on weather conditions.
- d. Effects on atmospheric constituent.

11. Briefly describe about the impacts of carbon monoxide on human health.

At lower doses, they can impair concentration and neurobehavioral function whereas in

higher doses they can cause heart pain and even death. When inhaled it has the ability to combine with haemoglobin of blood and reduce its ability in transfer of oxygen to the brain, heart, and other important organs. But carboxyaemoglobin contents of blood depend on the CO contents of the air inhaled, time of exposure and the activity of the person inhaling. It is particularly dangerous to babies and people with heart disease.

12. How air pollution can be controlled at source?

- a. Proper use of the existing equipment
- b. Change in process.
- c. Modification or Replacement of equipments.
- d. Installation of controlling equipments.

13. Define water pollution.

Water pollution may be defined as the alteration in physical, chemical and biological characteristics of water which may cause harmful effects on humans and aquatic life.

14. What are the effects of inorganic substances in water?

- a. Makes the water unfit for drinking and other purposes.
- b. Corrosion of metals exposed to such waters.
- c. Causes skin cancers, damages to spinal, CNS, liver and kidneys.
- d. Reduces crop yield.

15. How do the nutrients from agricultural fields affect the watershed?

Enrichment of nutrients (Eutrophication) from surrounding watershed affects the penetration of light through the water, causing damage to the characteristic of water and aquatic life.

16. Define soil pollution.

Soil pollution is defined as the contamination of soil by human and natural activities which may cause harmful effects on living beings.

17. Define marine pollution.

Marine pollution is defined as the discharge of waste substance into the sea resulting in harm to living resource, hazards to human health, hindrance to fishery and impairment of quality for use of sea water.

18. Define noise pollution.

Noise pollution is defined as the unwanted, unpleasant or disagreeable that causes discomfort for all living beings.

19. What is the cause of noise pollution?

- a. Road traffic noise
- b. Air traffic noise
- c. Rail traffic noise
- d. Domestic noise
- e. Industrial noise
- f. Incompatible land use.

20. Define thermal pollution.

Thermal pollution is defined as the addition of excess of undesirable heat to water that makes it harmful to man, animal or aquatic life or otherwise causes significant departures from the normal activities of aquatic communities in water.

21. What are solid wastes?

The wastes generated and discarded from human and animal activities that are normally solid are called as solid wastes.

22. What are solid know about on –site handling?

The activities involved in handling of solid wastes, at the point of generation, until they are placed in the containers used for their storage before collection are called as on –site handling. Handling requires to move the filled containers to the collection point and to return the empty containers to the generation point for the next collection.

23. What is the purpose of on-site processing?

On-site processing of solid wastes is used to recover the reusable materials from the solid wastes. This process also helps in reducing the volume of solid wastes or altering the physical form of the solid wastes.

24. What are the types of municipal solid wastes collection system?

1. Hauled container.
2. Stationary container systems.

25. What are the main purposes of processing techniques used in solid waste management?

- a. To improve the efficiency of solid waste management systems
- b. To recover the usable materials for reuse.
- c. To recover conversion products and energy.

26. List out the techniques of processing of solid wastes.

- a. Compaction (Mechanical volume reduction)
- b. Incineration (Chemical volume reduction)
- c. Shredding (Mechanical size reduction)
- d. Component separation
- e. Drying and Dewatering (Moisture content reduction).

27. Name some of the mechanical separation methods of solid wastes?

- a. Air separation
- b. Magnetic separation
- c. Screening

28. List out the three types of system used in the collection of wastewater.

- a. Separation System
- b. Combined System
- c. Partially Separate system.

29. What is the main objective of sludge digestion?

The main objective of sludge digestion is to break the organic matter of the sludge into liquid and simple compounds which are stable and unfold in nature.

30. How can you define hazardous wastes?

Wastes that create danger to the living community, immediately or over a period of time, are called as hazardous wastes.

31. What are biomedical wastes?

Biomedical wastes are defined as any solid, semi solid or liquid waste including its containers and any intermediate product which are generated during diagnosis, treatment or immunization of human being/ animals or in production and testing of biological parts.

32. Can you list out some of the benefits of pollution prevention?

- a. Minimizes health risks.
- b. Reduces the production of pollutants to a minimum or eliminates them.
- c. Accelerates the reduction or elimination of pollutants.
- d. Helps avoid transferring pollutants from one medium to another, thereby preventing diffusion in the environment.
- e. Helps promote a more effective use of energy, materials and resources.

UNIT III – NATURAL RESOURCES

1. Define renewable resources.

The renewable resources are those resources which have the inherent capacity to reappear, or replenish themselves by quick recycling, reproduction, and replacement within a reasonable time, and to maintain themselves. Example; air, water, soil (land), and plants, and animals

2. Define non-renewable resources.

The non-renewable resources are those that do not have the ability for recycling and replacement within a reasonable period of time.

Example; minerals, coal oil, natural gas, ground water

3. List some of the renewable energy sources.

- a. Solar energy
- b. Wind energy
- c. Hydro energy
- d. Geo-thermal energy
- e. Ocean thermal energy

4. Define deforestation.

Deforestation is the process of removal of (or) elimination of forest resources due to many natural or man-made activities. In general, deforestation means destruction of forests.

5. What is mining?

Mining is the process of extracting mineral resources and fossil fuels like coal from the earth. These deposits are found in the forest region and any operation of mining will naturally affect the forest. Mining operation requires removal of vegetation along with underlying soil mantle.

6. Define drought.

Drought is nothing but scarcity of water, which occurs due to inadequate rainfall, late arrival of rains and excessive withdrawal of ground water.

7. Define food resources.

Food is an essential requirement for the human survival. Each person has minimum food requirement, the main components of food are carbohydrates, fats protein, minerals, and vitamins.

8. Uses of forest.

- a. Wood used as fuel.
- b. Various industries. Used raw materials as pulp ,paper, board , timber
- c. Many plants are utilized in preparing medicines and drugs
- d. Forests products, like gums resins, dyes.

9. Types of mining. a.

surface mining :

It involves mining of minerals from the shallow deposits.

b. underground mining:

It involves mining of minerals from deep deposits.

10. What is hydrological cycle?

The water from various water bodies gets evaporated by the solar energy, and falls again on the earth in the form of rain or snow and enter into the living organisms and plants and ultimately returns to the ocean .this process is called hydrological cycle.

Clouds →water (ocean, lake, river) →living organisms &plants

11. Define floods.

A flood is an overflow of water, whenever the magnitude of flow of water exceeds the carrying capacity of the channel within its banks.

12. Cases of flood.

- a. Heavy rainfall, melting of snow (ice), sudden release of eater from dams, often causes floods in the low-lying coastal area.
- b. Prolonged downpour can also cause the over-flowing of lakes and rivers resulting into floods.
- c. The removal of dense and uniform forest cover over the hilly zones leads to occurrence of floods.

13. List some of the food resources available in the world.

Major food sources available in the world to cater the human's hunger are 12 types of seeds and grains, 3 root crops, 20 common fruits and vegetables, 6 mammals, 2 domestic fowl, few fishes and other forms of marine life etc.

14. Define Traditional Agriculture.

Traditional Agriculture can be classified as Traditional Subsistence agriculture and Traditional Intensive agriculture. Traditional Subsistence agriculture produces enough crops or live stock for a farm family's survival and in good years, a surplus to sell or put aside for hard times. In Traditional Intensive agriculture farmers increase their inputs of human and draft labour, fertilizer, and water to get a higher yield per area of cultivated land to produce enough food.

15. Write the advantages and disadvantages of petroleum as a energy resources?

As a source of energy petroleum has many advantages

- a. It is relatively cheap to extract and transport
- b. It requires little processing to produce desired products and
- c. It has relatively high net and useful energy yield.

However it has certain disadvantages also,

- a. Produces Environmental pollution
- b. Oil spills, in ocean cause water pollution and is expensive to clean up.

16. Write short notes on Tidal energy.

Tides, the alternate rise and fall of sea water possess lot of energy. The identified tidal power potential in India is around 9000 MW. Currently France, Russia, china and Canada are effectively utilizing the tidal energy to produce 2 to 3% of their energy demand.

17. Define Soil Erosion.

Soil erosion is the movement of soil components, especially surface litter and top soil, from one place to another. The two main movers are flowing water and wind.

18. List some ways to protect soil.

- a. When the buildings are constructed, all the trees shall be saved.
- b. Setting a composite bin and it shall be used for producing mulch and soil
- c. Conditioner for yard and garden plantso organic methods can be used for growing vegetables.
- d. Strictly enforcing laws and policies that minimize soil erosion, salt buildup and water logging.

19. What is equitable use of resources?

The Equitable use of resources is a concept that deals with the rational use of resources so that a harmony between man's resource requirement and its availability can be established.

20. Define drought.

Drought is nothing but scarcity of water which occurs due to inadequate rainfall, late arrival of rains and excessive withdrawal of ground water.

21. Give the classification of Mineral Resources.

Energy resources(coal, oil, natural gas, uranium, and geothermal energy; metallic mineral resources (iron, copper and aluminum) and nonmetallic minerals resources (salt, gypsum and clay, sand, phosphates, water, and soil).

22. Write short note on blue baby syndrome.

When the nitrogenous fertilizers are applied in the fields they leach deep into the soil the contaminate the ground water. the nitrate concentration in the water gets increased. When the nitrate concentration exceeds 25mg /lit they cause series health problems called blue baby syndrome this disease affects infants and leads even to death.

23. Define energy.

Energy may be defined as any property, which can be converted into work (or) the capacity to do work.

24. Write short notes on petroleum gas.

It is the mixture of three hydrocarbons butane, propane and ethane. The main constituent of petroleum gas is butane. The above gases are in gaseous state in ordinary pressure but they can be liquefied under high pressure. So it is called as LPG. (Liquefied petroleum Gas).

A domestic cylinder contains 14 kg of LPG. A strong smelling substance called ethyl mercaptan is added to LPG gas cylinder to help in the detection of gas leakage.

25. List some of the renewable energy sources.

- a. Solar energy
- b. Wind energy
- c. Hydro energy
- d. Geo-thermal energy
- e. Ocean thermal energy
- f. Biogas

26. Define wind energy.

Moving air is called wind. Energy recovered from the force of the wind is called wind energy. The energy possessed by wind because of its high speed. The wind energy is harnessed by making use of wind mills.

27. What is ote?

There is often large temperature difference between the surface level and deeper level of the tropical oceans. This temperature difference can be utilized to generate electricity. The energy available due to the difference in temperature of water is called ocean thermal energy.

28. Define geo thermal energy.

Temperature of the earth increases at a rate of 20-75°C per km, when we move down the earth surface. High temperature and high pressure steam fields.

29. Define LPG.

The petroleum gas obtained during the cracking and fractional distillation can be easily converted into liquid under high pressure called as LPG. LPG is colorless and odorless gas. But during bottling some mercaptans is added, which produces bad odour.

30. What is land degradation?

Land degradation is the process of deterioration of soil or loss of fertility of the soil.

UNIT IV– SOCIAL ISSUES AND THE ENVIRONMENT

1. State the declaration about the sustainable development.

The Rio declaration states that, “human beings are at the center or concern for sustainable development. They are entitled to a health and productive life in harmony with the nature. Rvery generation should leave air. Water and soil resources without any pollution as pure as it came to the Earth.”

2. Define sustainable development.

Sustainable development is defined as meeting the needs of the preset without compromising the ability of future generations to meet their own needs.

3. What are the three important components of sustainable development?

The three important components of sustainable development are

- a. Economic development (like industrial development, creating job opportunities, utilization of natural resources for developing the quality of life)
- b. Community development (providing food, shelter, cloth, education, and other essentials for the human beings).
- c. Environmental protection (providing clear air, water and environment for the present and future generations and utilization of resources in a sustainable manner).

4. Define sustainable development indicators

Sustainable development Indicators (SDI) is various statistical values that collectively measure the capacity to meet present and future needs. SDI will provide information crucial to decisions on national policy and to the general public.

5. What are the uses of sustainable development indicators?

The indicators are used by decision makers and the policy makers at all levels in order to monitor the progress towards attaining sustainable development. These are also used to increase focus on the sustainable development.

6. Define sustainability.

Sustainability can be defined as the ability of a society or ecosystem to continue functioning into the indefinite future without being forced into decline through complete loss of its strength or overloading of key resources on which that system depends.

7. Define resistance stability and resilience stability.

Resistance stability is the ability of a system to remain stable in the face of stresses and Resilience stability of the system to recover from the disturbance occurred due to the activities happened.

8. Define urbanization.

Urbanization is defined as „the process movement of human population from rural areas to urban areas in search in search of better economic interests with better education, communication, health, civic facilities and other day to day needs.

9. What are problems or discomforts faced by rural people?

- a. Lack of modernization of agricultural sector.
- b. Lack of job opportunities.
- c. Poor life style.
- d. Poor health facilities Poor education facilities.
- e. Poor transportation facilities.
- f. Poor availability of energy.

10. What are the uses of energy in urban areas?

Energy is used in an urban area for the following.

- a. For industrial activities
- b. For transportation
- c. For water apply
- d. For building & commercial use
- e. For cleaning of pollutants
- f. For essential services.

11. Define water conservation.

The process of saving water for future utilization is known as water conservation.

12. Define rain water harvesting.

Rain water harvesting is a technique of capturing and storing of rainwater for further utilization.

13. Define watershed.

Water shed is defined as the land area from which water drains under the influence of gravity into a stream, lake, reservoir or other body of surface water.

14. What do you know about watershed?

A watershed is defined as the geographic area from which water in a particular stream, lake or estuary originates. It includes entire area of land that drains into the water body. It is separate from other system by high points in the area such as hills or slopes.

15. What is watershed management?

Watershed management is a process aimed at protecting and restoring the habitat and water resources of a watershed, incorporating the needs of multiple stakeholders.

16. What are the impacts of human activities on watershed?

- a. Alteration of water course
- b. Addition of pollution sources
- c. Urbanization
- d. Securing of channels.

17. What are the two important principles of watershed management?

The two important principles of watershed management are:

- a. To preserve the environment, and
- b. To use the most cost-effective means to achieve this goal.

18. Name some of the factors causing relocation of people.

- a. Development activities
- b. Natural and man-made disasters
- c. Conservation initiatives.

19. Define environmental ethics.

Environmental ethics refers to the issues, principals and guidelines reading to human interactions with their environment.

20. Define resettlement.

Resettlement is defined as the process of simple relocation or displacement of human population without considering their individual, community or societal needs.

21. Define rehabilitation.

Rehabilitation is defined as the process of replacing the lost economic assets, rebuilding the community system that have been weakened by displacement, attending to the psychological trauma of forced separation from livelihood.

22. What are the factors that influence climate change on the earth?

Climate change on the earth is influenced by the following factors.

- a. Variations in the Earth's orbital characteristics.
- b. Atmospheric carbon dioxide variations.
- c. Volcanic eruptions
- d. Variations in solar output.

23. List out any four effects of climate change.

Mean sea level is increased on an average of around 1.8mm per year. Many ecosystems of the world have to adapt to the rapid change in global temperature. The rate of species extinction will be increased. Human agriculture, forestry, water resources and health will be affected.

24. Define green house effect.

The green house effects may be defined as the progressive warming up of the earth's surface due to blanketing effect of manmade CO₂ in atmosphere.

25. Define global warming.

The increased the inputs of CO₂ and other green house gases into the atmosphere from human activities will enhance the earth's natural green house effects of raising the average global temperature of the atmosphere near the surface. This enhanced the green house effect is called warming.

26. How can global warming are controlled.

- a. Reduction in consumption of fossil fuel such as coal and petroleum.
- b. Use of biogas plants.
- c. Use of nuclear power plants.
- d. Increasing forest cover.
- e. Use of unleaded petrol in automobiles.
- f. Installation of pollution controlling devices in automobiles and industries.

27. Define acid rain.

Normally rain water is always slightly acidic because of the fact that CO₂ present in the atmosphere gets dissolved on it. Because of the presence the of SO₂ and NO₂ gases as pollutants in the atmosphere, the pH of the rain water is further lowered. This type precipitation of water called acid rain or acid deposition.

28. List any four impacts of acid rain.

- a. Both dry and wet deposition of sulphur dioxide significantly increases the rate of corrosion of lime stone, sand and marble.
- b. Forest tree population is effected by acid rain.
- c. Acid rain in combination with ozone may damage the wxy coating on leaves and needles. This may weaken or damage them and provide opportunities for disease to enter the tree.
- d. Acid rain may change the characteristics of soil and eventually pollute the streams and lakes.

29. Define wet deposition and dry deposition. Is there any difference in damage due to these two types of deposition?

Wet deposition refers to acidic rain, fog, and snow. As this acidic water flowers over and through the ground, it afferts plants and animals in many ways. Dry deposition refers to acidic gases and particles. About half of the acidity in the atmosphere falls back to earth through dry deposition. Both wet and dry deposition can cause the same damage.

30. How can we minimize the formation of acid rain?

- a. By reducing pollution from industries,

- b. By using other sources of energy,

31. Define ozone layer.

Ozone is a gas O₃ found thorough the atmosphere but most highly concentration in the stratosphere between 10 and 50 km above sea level, which it is known as the ozone layer.

32. Name any three most important types of CFC which are responsible for ozone depletion.

- a. Trichlorofluoromethene,
- b. Dichlorodifluoromethane,
- c. 1, 1, 2 Trichlorotrifluoroethane.

33. What are the harmful effects of ozone layer depletion on human beings?

- a. Reddening of skin in sun shine (Sun burn)
- b. Skin Cancer.
- c. Reduction in body's immunity to disease.
- d. Eye disorders like Cataracts and Blindness.

34. Define waste land.

Waste lands are defined as the lands which re unstable in ecologically and topographically with complete loss or its fertility status. In these types of lands the toxicity for the growth of crops or trees are developed due to environmental or anthropogenic problems.

35. What are the causes for formation of waste land?

- a. Deforestation.
- b. Desertification.
- c. Soil loss.
- d. Industrial pollution.

36. What is the need for waste land reclamation?

Population of the world is increasing at an alarming rate. This increases demand for food and demand of land for shelter and other resources. The available land area should be properly utilized for making food for increasing population of the world.

UNIT V – HUMAN POPULATION AND THE ENVIRONMENT

1. How the population problem in India is analyzed?

India's population problem may be viewed from three aspects

- a. The absolute size of population.
- b. The rate of growth of the population.
- c. The age structure of the population.

2. What is population explosion?

Population explosion means the rapid population growth which is unexpected and unimaginable. The graph of recent population growth is referred to as a „J“ curve as it follows the shape of that letter, starting out low and skyrocketing straight up.

3. List the effect of population explosion.

- a. Increased consumption of resources available in the environment and depletion of the same quickly.
- b. Due to over –consumption of natural resources, the environment gets deteriorated and polluted.
- c. There will be desertification, deforestation, soil erosion, loss of fertility and poor productivity.
- d. Mass poverty, poor per-capital availability of food for consumption and prevalence of disease on large scale.
- e. Rapid urbanization resulting in growth of slums in cities and towns.

4. Define wellness.

Wellness is a state of optional well being. Wellness emphasizes each individual’s responsibility for making decisions that will lead not only to the prevention for disease but to the promotion of a high level of health.

5. Name some health related fitness components.

- a. Muscular strength and endurance.
- b. Flexibility.
- c. Body composition.
- d. Cardio-vascular endurance.

6. Define Demography.

It refers to the science of dealing with the study of size, composition and territorial distribution of population; it includes study of natality, fertility, mortality, migration, and social mobility.

7. What is vital statistics?

Vital statistics are referred to systematically collected and compiled data relating to vital events of life such as birth, death, marriage, divorce, adoption, etc.

8. Name the fundamental rights of an Indian citizen.

- a. Right to equality
- b. Right to freedom of Speech and Activity
- c. Right against Exploitation
- d. Right to Freedom of Religion
- e. Cultural and Educational Rights
- f. Right to Constitutional Remedies.

9. Write short notes on common property resources.

Our environment has a major component that does not belong to individuals. There are several commonly owned resources that all of us use as a community. The water that nature recycles, the air that we all breathe, the forests and grasslands which maintain our climate and soil, are all common property resources

10. What is HIV?

HIV stands for Human Immuno-deficiency Virus and is a virus that can damage the body’s defence system so that it cannot fight off certain infections.

11. What is AIDS?

AIDS stands for (Acquired Immuno Deficiency Syndrome). An HIV infected person receives a diagnosis of AIDS after developing one of the AIDS indicator illness, A positive HIV test result does not mean that person has AIDS. A diagnosis of AIDS is made by a physician using certain clinical criteria (Eg: AIDS indicator illnesses).

12. What is opportunistic infection?

Infection with HIV can weaken the immune system to the point that it has difficulty fighting off certain infections. These types of infections are known as “opportunistic infections” because they take the opportunity to weaken the immune system which causes illness of the body.

13. List the means of HIV transmission.

There are four main ways in which HIV can be passed on:

- a. By having vaginal, anal or oral sex without a condom with someone who has HIV.
- b. By using needles, syringes or other drug-injecting equipment that is infected with HIV.
- c. From a woman with HIV to her baby (before or during birth) and by Breast feeding.
- d. By receiving infected blood, blood products or donated organs as part of medical treatment.

14. Name some tests available to find HIV infection.

- a. In addition to the EIA or ELISA and Western blot, other tests now available include:
- b. Radio Immuno Precipitation Assay (RIPA)
- c. Dot –blot immuno binding assay
- d. Immuno fluorescence assay
- e. Nucleic acid testing
- f. Polymerase Chain Reaction (PCR)

15. List the special features of Comprehensive programme on women and child welfare.

- a. Personality.
- b. Reduction of Deprivation.
- c. Co-ordinational Effectivity.
- d. Maternity and Motherhood.

16. Name some applications of IT in health.

Apart from helping in the administration of hospitals, IT is playing a key role in the health industry. On the, medical care, the IT has varied applications right from the diagnosis, where there are latest tools like CT scans, Ultrasound Sonography etc. Which use It as their basis for diagnosis of ailments. Most of the

ICU’s (Intensive Care Units) are now using computers to monitor the progress and condition of the patient, undergoing treatments. Apart from this, with help of IT, expert opinions from doctors away from the place can be sought with help of IT tools like video conferencing etc. Apart from this can be used in the analysis and research on various potential medicines /drugs to be used in medical treatments.

17. List the applications of IT in environment.

- a. Remote Sensing
- b. Geographic Information System (GIS)
- c. Global Positioning System(GPS)
- d. Meterology

18.What is information?

The term “information” has been defined by Eliahu Hoffinan as: “an aggregate (Collection and accumulation) of statements, or facts or figures which are conceptually by way of reasoning, logic, ideas, or any other mental “mode operation” interrelated/connected.

19. Write short notes on common property resources.

Our environment has a major component that does not belong to individuals. There are several commonly owned resources that all of us use as a community. The water that nature recycles, the air that we all breathe, the forests and grasslands which maintain our climate and soil, are all common property resources.

20. What is health?

Health is considered as a quality of life that enables the individual to live most and serve best.

21. Define dilution factor.

The dilution factor is defined as the ratio of the amount of river water to the amount of the sewage.

Environmental Science and Engineering
Unit I- Environment, Ecosystems and
Biodiversity

Part- B(16 marks)

- 1.(i) Explain the various threats to Biodiversity.
(ii) What are the causes for loss of biodiversity?
- 2.(i) What is meant by value of biodiversity? Explain different values of biodiversity.
(ii) Explain In-situ and Ex- situ conservation of biodiversity.
- 3)(i) Explain the role of biodiversity of global, national and local levels.
(ii) Describe the term hot spot in biodiversity
- 4.(i) Briefly explain the energy flow through ecosystem.
(ii) Describe the types, characteristic features, structures and function of Forest ecosystem,
- 5.(i) Discuss the concept of ecological succession.
(ii) Briefly explain the structural and functional components of an ecosystem.
6. (i) Write a note on endangered and endemic species.
(ii) Explain the scope and significance of environmental studies.
- 7.i) Name and briefly describe two hot spots of biodiversity in India
ii) Describe the types, characteristic features, structures and function of aquatic ecosystem.
8. i) Discuss the major features, structure and composition of grassland ecosystem.
ii) Discuss the structure and composition of fresh water ecosystem.
- 9 i) What are the cycles in ecosystems? Describe carbon cycle.
ii) Give the classification and explain various levels of biodiversity.
- 10 i) Discuss the structure and function of desert ecosystem.
ii) Discuss the biogeochemical cycle in the ecosystem.

Unit II -Environmental Pollution

Part- B (16 marks)

- 1.(i) Explain the methods of disposal of municipal solid waste.
(ii) Write a note on disposal of radioactive wastes.
2. (i) Explain the causes, effects and control measures of water pollution.
(ii) Explain the effects of nuclear and Radiation pollution.
3. (i) Explain the causes, effects and control measures of marine pollution.
(ii) Write a short note on disaster management.
4. (i) Discuss the major soil pollution and their impact.
(ii) What is thermal pollution and explain its effects.
5. (i) Discuss about the significance of hazardous waste management.
(ii) Discuss the major air pollutants and their impact.
6. (i) Describe the sources, effects and control of noise pollution.
(ii) Discuss the role of individual in preventing pollution.
- 7(i) What is acid rain? How it is formed? Give its effects.
(ii) What is ozone? How it is depleted? Enumerate the effects of ozone layer depletion.
- 8(i) Discuss in detail about waste water treatment process.
(ii) Discuss the role of individual in prevention of pollution.
- 9(i) Draw the flow chart for solid waste management and give the sources of it.
(ii) Discuss in detail about the Bhopal gas tragedy and Chernobyl nuclear disaster.
10. (i) How is the water treated by zeolite process.
(ii) Explain in detail about the demineralization process.

Unit III- Natural Resources

Part- B (16 marks)

- 1.(i) Discuss briefly the ill-effects of deforestation.
(ii) What are the effects of modern Agriculture?
2. (i) What are the causes of soil erosion & deforestation? Explain in detail.
(ii) Discuss the consequences of overutilization of surface and Ground water.
3. (i) Write the effects of extracting and using mineral resources

- (ii) Discuss the causes of land degradation.
- 4. (i) Explain how the alternate energy sources play an important role in environment impact.
- (ii) Write a note on solar production of electricity.
- 5. (i) Explain the role of an individual in environment protection.
- (ii) Explain the methods of harnessing wind and ocean thermal energy.
- 6. (i) Compare nuclear power with coal power.
- (ii) Write briefly on the Hydrologic Cycle.
- 7. (i) What are the ecological benefits of forest?
- (ii) Environmental damage caused by mining last longer after the mine has closed – explain.
- 8. (i) Discuss the effects of dams on forest and tribal people.
- (ii) Explain the benefits and problems of constructing dam.
- 9. (i) Explain any two water conflicts confining to India.
- (ii) Explain various steps involved in anaerobic digestion.
- 10. (i) Explain the production and uses of biogas
- (ii) Write notes of bioconversion of pollutants and degradation of proteins.

UNIT IV- Social Issues and the Environment

Part- B (16 marks)

- 1. (i) Explain the agenda for sustainable development.
- (ii) Explain the need and strategy of water conservation.
- 2. (i) Explain wasteland reclamation.
- (ii) Discuss nuclear accidents and holocaust.
- 3. (i) Define environmental ethics. Mention the problems and solutions related to environment
- (ii) Explain briefly on the Indian Environmental Acts.
- 4. (i) Explain watershed management.
- (ii) Explain rain water harvesting.
- 5. (i) Explain resettlement and rehabilitation issues.
- (ii) Explain the urban problems related to energy.
- 6. (i) Give the principles of Green chemistry.
- (ii) How is environment protected through Legislation? Explain.
- 7. (i) What is consumerism? Mention the objectives and factors affecting consumerism.
- (ii) Explain in detail on environment and water act.
- 8. (i) What is a biomedical waste? Give the steps involved in management of biomedical waste.
- (ii) What is an eco mark? Give the objectives of it. Give the criteria for awarding eco mark.
- 9. (i) Give the drawbacks of environmental laws.
- (ii) What is an earthquake? Enumerate its effects. Mention the methods to mitigate the disaster.
- 10. (i) Explain various emergency management tips to be under taken in case of any disaster.
- (ii) State different natural calamities and explain one in detail.

UNIT V- Human Population and the Environment

Part- B (16 marks)

- I. (i) Explain briefly the population explosion.
- (ii) Explain the role of IT in environment and human health. ' '
- 2. (i) Explain in detail about population growth.
- (ii) Explain in detail about family welfare programme, training and development.
- 3. (i) Explain the value of education.
- (ii) Explain briefly on Human rights.
- 4. (i) Explain HIV/AIDS.
- (ii) Explain the various policies and programmes for women and child development.
- 5. (i) Write short notes on a) women and child welfare b) Human rights c) Value education.
- (ii) Outline the various family welfare plans in the post independent India.
- 6. (i) Explain the role of NGOs in environmental protection and health.
- (ii) Discuss about the environmental and social problems faced by a large Indian city like Chennai.
- 7. (i) Draw a typical population pyramid of a developing country and discuss how it differs from that of a developed country.

- (ii) Deterioration of environment leads to Deterioration of human health. – Justify.
- 8(i) Write in detail on implementing family planning programme.
(ii) Discuss the factors influencing family size.
- 9(i) Population explosion leads to environmental deterioration. Explain.
(ii) Write a note on various methods of family planning.
- 10(i) With a neat diagram explain the variation in population growth among various nations.
(ii) What is EIA? Give the objectives, benefits and process of EIA.

BASIC ELECTRICAL AND ELECTRONICS ENGINEERING

2 MARKS
UNIT I- ELECTRICAL CIRCUITS & MEASUREMENTS

1. What is meant by linear and nonlinear elements?

Linear element shows the linear characteristics of voltage V vs current. Nonlinear element the current passing through it does not change linearity with the linear change in applied voltage at a particular frequency.

2. What is meant by active and passive elements?

If a circuit element has the capability of enhancing the energy level of a signal passing through it is called an active element. Passive elements do not have any intrinsic means of signal boosting.

3. What is meant by unilateral and bilateral elements?

If the magnitude of the current passing through an element is affected due to change in the polarity of the applied voltage is called unilateral elements. If the current magnitude remains the same even if the applied EMFs polarity is changed is called bilateral elements.

4. Define Ohms Law.

The potential difference across any two ends of a conductor is directly proportional to the current flowing between the two ends provided the temperature of the conductor remains constant.

5. Mention the disadvantages of Ohm's Law.

- It does not apply to all non metallic conductors
- It also does not apply to non linear devices such as zener diode, vacuum tubes etc.
- It is true for metal conductors at constant temperature. If the temperature changes the law is not applicable.

6. State voltage division rule.

Voltage across a resistor in a series circuit is equal to the total voltage across the series elements multiplied by the value of that resistor divided by the total resistance of the series elements.

7. State current division rule.

The current in any branch is equal to the ratio of the opposite parallel branch resistance to the total resistance value, multiplied by the total current in the circuit.

8. What is transient state?

If a network contains energy storage elements, with change in excitation, the current and voltage change from one state to other state the behavior of the voltage or current when it is changed from one state to another state is called transient state.

9. What is transient time?

The time taken for the circuit to change from one steady state to another steady state is called transient time.

10. What is transient response?

The storage elements deliver their energy to the resistances; hence the response changes with time, get saturated after sometime, and are referred to the transient response. The transient response is defined as the response or output of a circuit from the instant of switching to attainment of steady state

11. Define time constant of RLC circuit.

The time taken to reach 63.2% of final value in a RL circuit is called the time constant of RL circuit. **Time constant=L/R**

12. Define time constant of RC circuit.

The time taken to reach 36.8% of initial current in an RC circuit is called the time constant of RC circuit. **Time constant= RC**

13. What is meant by natural frequency?

If the damping is made zero then the response oscillates with natural frequency without any opposition, such a frequency is called natural frequency of oscillations.

14. Define damping ratio.

It is the ratio of actual resistance in the circuit to the critical resistance.

15. Write down the few applications of RL, RC, RLC circuits.

- Coupling circuits
- Phase shift circuits
- Filters
- Resonant circuits
- AC bridge circuits
- Transformers

16. Define apparent power.

The apparent power is defined as the product of magnitude of voltage and magnitude of current.

17. What is power factor and reactive power?

The power factor is defined as the cosine of the phase difference between voltage and current. **Power factor= $\cos\phi$**

The reactive power of the circuit is defined as the sine of the phase angle.

$$\text{Reactive power} = \sin\phi$$

18. Give the line and phase values in star connection.

The relation between line and phase voltage in star connection is $E_L = \sqrt{3}E_{ph}$. The relation between line current and phase current in a star connection is $I_L = I_{ph}$.

19. Give the line and phase values in delta connection.

The relation between line voltage and phase voltage in a delta connection is $E_L = E_{ph}$. The relation between line current and phase current in delta connection is $I_L = \sqrt{3}I_{ph}$.

20. Write few methods available for measuring in 3-phase load.

- One wattmeter method
- Two wattmeter method
- Three wattmeter method

21. Mention the functions performed by the measurement system.

The functions performed by the measurement system are

- Indicating function
- Recording function
- Controlling function

22. Why the PMMC instruments are not used for a.c measurement?

When the PMMC instruments are connected to a.c, the torque reverse as the current reverses and the pointer cannot follow the rapid reversals. Hence the deflection corresponding to mean torque is zero thus making the PMMC instrument not suitable for a.c measurements.

23. What is the basic principle PMMC instrument?

A current carrying coil is placed in the magnetic field experiences a force proportional to the current it carries.

24. For which type of measurements PMMC devices are suitable.

These are suitable only for D.C. In A.C torque produced on coil is reversing which cannot give accurate reading.

25. What are the sources of error in DC voltage measurement?

- Friction in the moving system
- Heat generated changes the resistance of the working coil.
- The ageing if the magnet and control spring

26. List the possible causes of errors in moving iron instruments.

- Hysteresis error
- Temperature error
- Stray magnetic field error
- Frequency error
- Eddy current error

UNIT II- ELECTRICAL MACHINES

1. Mention the difference between core and shell type transformers.

In core type the windings surround the core considerably and in shell type the core surround the winding.

2. What is the purpose of laminating the core in transformers?

To reduce eddy current loss.

3. Does the transformer draw any current when secondary is open? Why?

Yes, it (primary) will draw the current from the main supply in order to magnetize the core and to supply iron and copper losses on no load. There will not be any current in the secondary since secondary is open.

4. Define voltage regulation of a transformer.

When a transformer is loaded with a constant primary voltage, the secondary voltage decreases for lagging power factor load, and increases for leading Pf load because of its internal resistance and leakage reactance. The change in secondary terminal voltage from no load to full load expressed as a percentage of no loads or full load voltage is termed as regulation.

% regulation down = $(\text{No load } V_2 - \text{Full load } V_2) / \text{No load } V_2 * 100$

% regulation up = $(\text{No load } V_2 - \text{Full load } V_2) / \text{Full load } V_2 * 100$

5. Define all day efficiency of a transformer.

It is the computed on the basis of energy consumed during a certain period, usually a day of 24 hrs.
 $\eta_{\text{all day}} = \text{output in kWh} / \text{input in kWh for 24 hrs.}$

7. Why transformers are rated in kVA?

Copper loss of a transformer depends on current and iron loss on voltage. Hence total losses depends on Volt- Ampere and not on the power factor. That is why the rating of transformers is in kVA and not in kW.

8. What are the typical uses of auto transformer?

(i) To give small boost to a distribution cable to correct for the voltage drop. (ii) As induction motor starters.

(iii) As furnace transformers

(iv) As interconnecting transformers

(v) In control equipment for single phase and 3 phase electric locomotives.

9. What are the applications of step-up and step-down transformers?

Step-up transformers are used in generating stations. Normally the generated voltage will be either 11 kV or 22 kV. This voltage is stepped up to 110 kV or 220 kV or 400 kV and transmitted through transmission lines. (In short it may be called as sending end). Step-down transformers are used in receiving stations. The voltage are again stepped down to 11 kV or 22 kV and transmitted through feeders. (In short it may be called as receiving end).

10. What are the two types of cores used? Compare them.

Transformers are classified according to their construction as,

(i) Core type

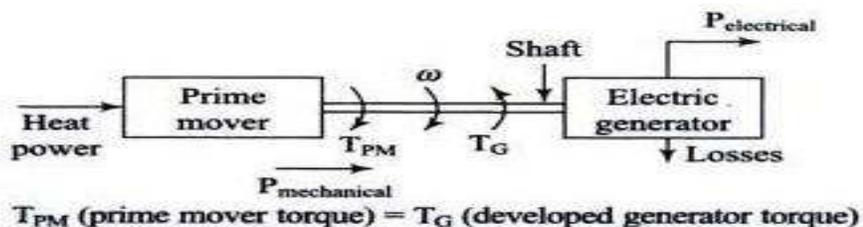
(ii) Shell type

(iii) Spira core type.

Spira core type is a latest transformer and is used in big transformers. In “core” type, the windings (primary and secondary) surround the core and in “shell” type, the core surrounds the windings.

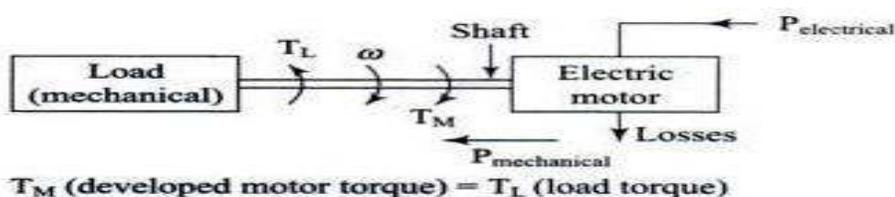
11. What do you meant by Electric Generator?

Electricity does not occur naturally in usable form and it also cannot be stored in usefully large quantities. Therefore, it must be generated continuously to meet the demand at all times. An efficient and convenient way to generate electric power is by conversion of mechanical power into electrical form in rotating device called Generator



12. Define Electric Motor.

The major use of electric energy is made by converting it back to run the wheels of industry as well as tiny household appliances. The electromechanical energy conversion process is a reversible one and simple adjustment of mechanical shaft and electrical conditions reverses the flow of power. In this mode of operation, the electromechanical device, in general called the electric machine, is known as the motor and the machine is said to be in the motoring mode.



13. What are all the main parts of a DC machine? (Oct – 97)

Stationary member called stator

Rotating member called rotor

Field winding wound on field poles to produce uniform magnetic flux

Armature winding used to interchange current with the external electric system depending upon the circuit conditions

Commutator – a mechanical rectifier

Brushes – used to collect the current

14. Write down the EMF equation of a DC Generator (April – 98)

$$E_a = (\phi Z n P) / (60 A) \text{ Volts}$$

Where ϕ - the magnetic flux/pole in Wb

n - The armature speed in rpm

Z - Total armature conductors

A – Number of parallel paths

P – Number of poles

15. Write down the torque equation of a DC motor? (Nov 2010)

$$T = K_a \phi I_a \text{ Nm}$$

Where $K_a = (1/2\pi)Z(P/A)$

ϕ - Magnetic flux / pole in Wb

I_a – Armature current in amps

16. What is back emf in d.c motors?

As the motor armature rotates, the system of conductor come across alternate North and South Pole magnetic fields causing an emf induced in the conductors. The direction of the emf induced in the conductors. The direction of the emf induced is in the direction opposite to the current .As this emf always opposes the flow of current in motor operation it is called back emf.

17. Describe the main parts of a single phase induction motor.

The single phase induction motor has two main parts. They are the stator and the rotor. The stator is made up of laminated sheets and has slots on the inner periphery to carry the stator winding or the main winding. The main winding is excited by a single phase ac supply. The rotor is of squirrel cage type which has number of conducting bars short circuited at the two ends using end rings.

18. Why is the single phase induction motor not self starting?

The single phase induction motor is not self starting because of the absence of a rotating magnetic field. Since it is operated on single phase, there exists only one flux which is alternating in nature. But for the motor to start there should be a rotating magnetic field.

19. What are the types of single phase induction motors?

There are four main types of single phase induction motors. They are,

- a) Split phase induction motor
- b) Capacitor start induction motor
- c) Capacitor start capacitor run induction motor
- d) Shaded pole induction motor

20. Why is the starting torque of a capacitor start induction motor high, when compared to that of a split phase induction motor?

The starting torque of the single phase induction motor is directly proportional to the phase angle between the currents carried by the two windings, namely the main winding and the starting winding.

Split phase induction motor uses a highly resistive starting coil, which gives the phase angle nearly 90° . But in capacitor start induction motor, the phase angle obtained is greater than 90° . Hence the starting torque is still higher in this type.

UNIT III – SEMICONDUCTOR DEVICES & APPLICATIONS

1. Define Knee Voltage of a diode.

The minimum voltage at which the diode starts conducting and current starts increasing Exponentially is called knee voltage of a diode.

2. What is peak inverse voltage?

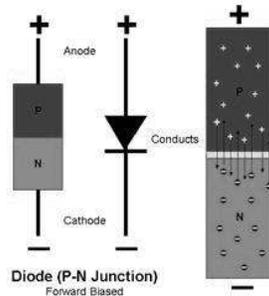
In reverse biased, opposite polarity voltage appears across diode. The maximum voltage which diode can withstand without breakdown is called peak inverse voltage.

3. Differentiate drift and diffusion current.

When a voltage is applied to a material, the free electrons move towards the positive of the battery. While moving they collide with the adjacent atoms and keep changing their directions randomly. In case of diffusion current, the external voltage is not required.

4. What is a PN Junction diode?

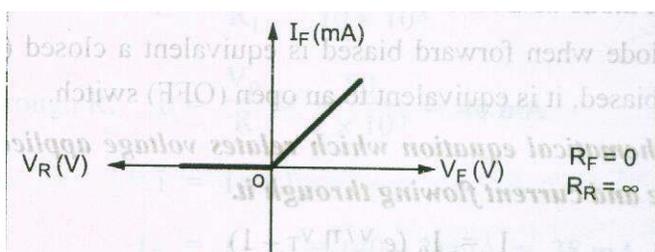
There are two electrodes each from p-type and n-type materials and due to these two electrodes; the device is called a diode. It conducts only in one direction.



5. Differentiate avalanche and zener breakdowns.

Zener Breakdown	Avalanche Breakdown
1. Breakdown occurs due to heavily doped junction and applied strong electric field.	1. Breakdown occurs due to avalanche multiplication between thermally generated ions.
2. Doping level is high.	2. Doping level is low.
3. Breakdown occurs at lower voltage compared to avalanche breakdown.	3. Breakdown occurs at higher voltage.

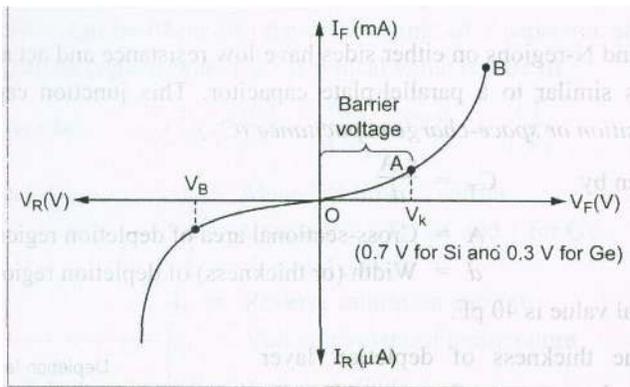
6. Draw the V- I characteristics of an ideal diode.



7. Differentiate between drift and diffusion currents.

Drift Current	Diffusion Current
1 . It is developed due to potential gradient.	1 . It is developed due to charge concentration gradient.
2 . This phenomenon is found both in metals and semiconductors.	2 . It is found only in semiconductors.

8. Draw the V-I characteristics of a practical PN diode.



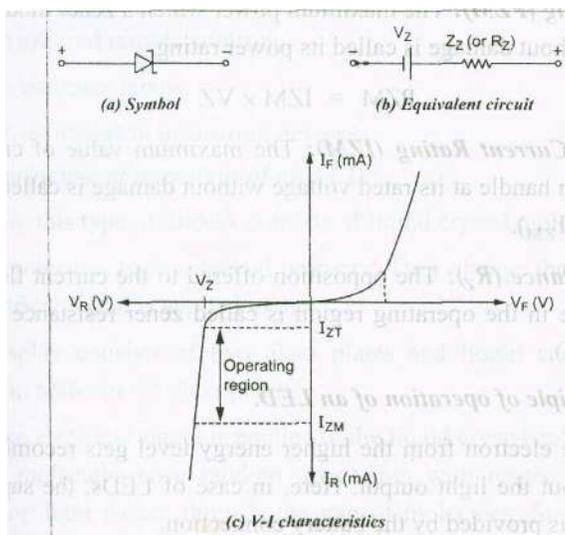
9. Define transition capacitance of a diode.

Transition Capacitance (C_T) or Space-charge Capacitance: When a PN-junction is reverse -biased, the depletion region acts like an insulator or as a dielectric.

10. Define diffusion capacitance of a diode.

Diffusion or Storage Capacitance (C_D): This capacitive effect is present when the junction is forward-biased.

11. Draw the V -I characteristics of a zener diode.



12. Define rectifier. Mention the types.

Rectifier: A rectifier is a circuit that converts AC into pulsing DC. It uses unidirectional conducting devices like PN diodes. Rectifiers are classified into two types based on the conduction of AC input.

They are:

- Half wave rectifier (HWR).
- Full wave rectifier (FWR).

13. Define rectifier efficiency.

It is defined as the ratio of DC power output to the applied AC power input

14. Define ripple factor of a rectifier.

The purpose of a rectifier is to convert AC into DC. But the pulsating output of a rectifier contains a DC component and an AC component, called ripple. The ratio of RMS value of AC components to the DC component in the rectifier output is called 'ripple factor'.

15. Define TUF of a rectifier.

Most of the rectifier circuits make use of transformer whose secondary feeds the AC power. The transformer rating is necessary to design a power supply. Transformer utilization factor (TF) is defined as the ratio of DC power delivered to the load to the AC power rating of transformer secondary.

16. Calculate β of a transistor when $\alpha = 0.98$

$$\beta = \alpha / (1 - \alpha) = 0.98 / (1 - 0.98) = 49$$

17. Among CE, CB and CC configurations, which one is the popular? Why?

The CE configuration is widely used because it provides both voltage gain as well as current gain greater than unity.

18. Name the operating modes of transistor?

Cut off

Active

Saturation

19. Mention the disadvantages of FET compared to BJT.

- FET is a voltage controlled device
- Less sensitivity to changes in applied voltage

20. Define pinch off voltage of a FET

The substrate is of p-type material on to which an n-type channel is epitaxially grown. A p-type gate is then diffused into the n-type channel.

21. What is MOSFET? Name its types.

- MOSFET is a metal oxide semiconductor field effect transistor
- Its types are depletion and enhancement type

22. Why FET is called voltage controlled device ?

The voltage applied between gate and source (VGS) controls the drain current Id. Therefore FET is called as voltage controlled device.

23. Give any two differences between E-MOSFET and D-MOSFET

S.N O	PARAMETERS	E-MOSFET	D-MOSFET
1.	Channel	Exists Permanently	Channel is physically absent
2.	Operation	Can be operated in depletion mode as well as enhance mode	Can only be operated in enhance mode

24. Compare JFET with BJT.

Parameter	BJT	JFET
Control element	Current controlled device	Voltage controlled device
Configurations	CE, CB and CC	CC,CG, CD

UNIT IV –DIGITAL ELECTRONICS

1. Define binary logic?

Binary logic consists of binary variables and logical operations. The variables are designated by the alphabets such as A, B, C, x, y, z, etc., with each variable having only two distinct values: 1 and 0. There are three basic logic operations: AND, OR, and NOT.

2. Convert (4021.2)₅ to its equivalent decimal.

$$(4021.2)_5 = 4 \times 5^3 + 0 \times 5^2 + 2 \times 5^1 + 1 \times 5^0 + 2 \times 5^{-1}$$

$$= (511.4)_{10}$$

3. Represent binary number 1101 - 101 in power of 2 and find its decimal equivalent.

$$N = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$$

$$= 13.625_{10}$$

4. Convert (634)₈ to binary.

6	3	4
110	011	100

Ans = (110 011 100)₂

5. Convert (9 B 2 - 1A) H to its decimal equivalent.

$$N = 9 \times 16^2 + B \times 16^1 + 2 \times 16^0 + 1 \times 16^{-1} + A (10) \times 16^{-2}$$

$$= 2304 + 176 + 2 + 0.0625 + 0.039$$

$$= 2482.1_{10}$$

6. What are the different classifications of binary codes?

- Weighted codes
- Non - weighted codes
- Reflective codes

- Sequential codes
- Alphanumeric codes
- Error Detecting and correcting codes.

7. Define logic gates?

Logic gates are electronic circuits that operate on one or more input signals to Produce an output signal. Electrical signals such as voltages or currents exist throughout a digital system in either of two recognizable values. Voltage- operated circuits respond to two separate voltage levels that represent a binary variable equal to logic 1 or logic 0.

8. Define duality property.

Duality property states that every algebraic expression deducible from the postulates of Boolean algebra remains valid if the operators and identity elements are interchanged. If the dual of an algebraic expression is desired, we simply interchange OR and AND operators and replace 1's by 0's and 0's by 1's.

9. State De Morgan's theorem.

De Morgan suggested two theorems that form important part of Boolean algebra. They are,

- The complement of a product is equal to the sum of the complements. $(AB)' = A' + B'$
- The complement of a sum term is equal to the product of the complements. $(A + B)' = A'B'$

10. Reduce A (A + B)

$$\begin{aligned} A(A + B) &= AA + AB \\ &= A(1 + B) [1 + B = 1] \\ &= A. \end{aligned}$$

11. Reduce A'B'C' + A'BC' + A'BC

$$\begin{aligned} A'B'C' + A'BC' + A'BC &= A'C'(B' + B) + A'B'C \\ &= A'C' + A'BC [A + A' = 1] \\ &= A'(C' + BC) = A'(C' + B) [A + A'B = A + B] \end{aligned}$$

12. What are the basic digital logic gates?

The three basic logic gates are

- AND gate, OR gate, NOT gate

13. Which gates are called as the universal gates? What are its advantages?

The NAND and NOR gates are called as the universal gates. These gates are used to perform any type of logic application.

14. Define Decoder?

A decoder is a multiple - input multiple output logic circuit that converts coded inputs into coded outputs where the input and output codes are different. A decoder is a combinational circuit that converts binary information from n input lines to a maximum of 2^n out puts lines.

15. What are the classifications of sequential circuits?

The sequential circuits are classified on the basis of timing of their signals into two types. They are,

- Synchronous sequential circuit.
- Asynchronous sequential circuit.

16. Define Flip flop and what are the different types of flip-flop.

The basic unit for storage is flip flop. A flip-flop maintains its output state either at 1 or 0 until directed by an input signal to change its state. There are various types of flip flops. Some of them are mentioned below they are, RS flip-flop, SR flip-flop, D flip-flop, K flip-flop, T flip-flop

17. Define registers.

A register is a group of flip-flops flip-flop can store one bit information. So an n- bit register has a group of n flip-flops and is capable of storing any binary information/number containing n-bits.

18. Define shift registers and write its types.

The binary information in a register can be moved from stage to stage within the register or into or out of the register upon application of clock pulses. This type of bit movement or shifting is essential for certain arithmetic and logic operations used in microprocessors. This gives rise to group of registers called shift registers. There are five types. They are,

- Serial In Serial Out Shift Register
- Serial In Parallel Out Shift Register
- Parallel In Serial Out Shift Register
- Parallel In Parallel Out Shift Register
- Bidirectional Shift Register

19. Define sequential circuit?

In sequential circuits the output variables dependent not only on the present input variables but they also depend up on the past history of these input variables.

20. Give the comparison between combinational circuits and sequential circuits.

Combinational circuits	Sequential circuits
Memory unit is not required	Memory unity is required
Parallel adder is a combinational circuit	Serial adder is a sequential circuit

21. Give the comparison between synchronous & Asynchronous sequential circuits?

Synchronous sequential circuits	Asynchronous sequential circuits
Memory elements are clocked flip-flops	Memory elements are either unlocked flip-flops or time delay elements.
Easier to design	More difficult to design

22. State the types of counter?

- Synchronous counter
- Asynchronous Counter

UNIT V- FUNDAMENTALS OF COMMUNICATION ENGINEERING

1. Define amplitude Modulation.

Amplitude Modulation is the process of changing the amplitude of a relatively high frequency carrier signal in proportion with the instantaneous value of the modulating signal.

2. Define Modulation index and percent modulation for an AM wave.

Modulation index is a term used to describe the amount of amplitude change present in an AM waveform. It is also called as coefficient of modulation.

Mathematically modulation index is

$$m = E_m / E_c$$

Where m = Modulation coefficient

E_m = Peak change in the amplitude of the output waveform voltage.

E_c = Peak amplitude of the unmodulated carrier voltage.

Percent modulation gives the percentage change in the amplitude of the output wave when the carrier is acted on by a modulating signal.

3. Explain in detail about FM modulators.

FM modulators are classified into two types. They are

1. Direct FM modulators
2. Indirect FM modulators

Direct FM modulator:

In direct frequency modulation, frequency of a constant amplitude carrier signal is directly proportional to the amplitude of the modulating signal at a rate equal to the frequency of the modulating signal. There are three common methods for producing direct frequency modulation: Varactor diode modulators, FM reactance modulators, and linear integrated circuit direct FM modulators.

Indirect FM modulator:

In indirect frequency modulation, phase of a constant amplitude carrier directly proportional to the amplitude of the modulating signal at a rate equal to the frequency of the modulating signal.

4. Explain in detail about FM demodulators.

FM demodulators are frequency dependent circuits designed to produce an output voltage that is proportional to the instantaneous frequency. Several circuits are used for demodulating FM signals. The most common are the slope detector, Foster Seeley discriminator, and ratio detector are forms of tuned circuit frequency discriminators.

5. Explain in detail about Ground wave propagation.

A ground wave is an electromagnetic wave that travels along the surface of earth. Therefore ground waves are sometimes called surface waves. Ground waves must be vertically polarized. Its advantages are

1. Ground waves are relatively unaffected by changing atmospheric conditions.
2. If the transmitted power is large enough, then ground wave propagation can be used to communicate between any two points in the world.

6. Explain in detail about Space wave propagation.

Space wave propagation of electromagnetic energy includes radiated energy that travels in the lower few miles of earth's atmosphere. Space waves include both direct and ground reflected waves. Space wave propagation with direct waves is commonly called line of sight transmission.

7. Explain in detail about Data modems.

The primary purpose of data modem is to interface computers, computer networks, and other digital terminal equipment to analog communication lines and radio terminals. Data modems are generally classified in to synchronous and asynchronous data modems.

Asynchronous Modems:

Asynchronous modems are used primarily for low speed data circuits. It use ASK or FSK.

Synchronous data modems:

Synchronous data modems are used for medium and high speed data transmission and use either PSK or QAM modulation.

8. Explain in detail about ISDN.

The Integrated Services Digital Network is a proposed network to provide voice, data, video, and facsimile information within the same network.

There are three basic types of channels available with ISDN. They are B channel: 64Kbps

D channel: 16 or 64 Kbps

H channel: 384,1536, or 1920 Kbps

9. Explain in detail about geosynchronous satellite. State its advantages and disadvantages.

Geo synchronous or geo stationary satellites are those that orbit in a circular pattern with an angular velocity equal to that of Erath. Geosynchronous satellites, have an orbital time of approximately 24 hours, the same as earth; thus geosynchronous satellites appear to be stationary as they remain in a fixed position in respect to a given point on earth.

10. Define satellite.

Satellite is a celestial body that orbits around a planet. In aerospace terms, a satellite is a space vehicle launched by humans and orbits earth or another celestial body.

11. State Kepler's first law.

Kepler's first law states that a satellite will orbit a primary body following an elliptical path.

12. State Kepler's second law.

Kepler's second law states that for equal time intervals of time a satellite will sweep out equal areas in the orbital plane, focused at the bary center.

13. State Kepler's third law.

The third law states that the square of the periodic time of orbit is proportional to the cube of the mean distance between the primary and the satellite.

14. Define orbital satellite.

Orbital satellites are also called as nonsynchronous satellite. Nonsynchronous satellites rotate around earth in an elliptical or circular pattern. In a circular orbit, the speed or rotation is constant however in elliptical orbits the speed depends on the height the satellite is above the earth

15. Define Geo synchronous satellite.

Geo synchronous or geo stationary satellites are those that orbit in a circular pattern with an angular velocity equal to that of Earth. Geosynchronous satellites have an orbital time of approximately 24 hours, the same as earth; thus geosynchronous satellites appear to be stationary as they remain in a fixed position in respect to a given point on earth.

16. Define apogee and perigee.

The point in an orbit which is located farthest from the earth is called apogee.

The point in an orbit which is located closest to earth is called perigee.

17. Define angle of inclination.

The angle of inclination is the angle between the earth's equatorial plane and the orbital plane of a satellite measured counterclockwise at the point in the orbit where it crosses the equatorial plane traveling from south to north.

18. What are the advantages of optical fiber communication? _ Greater information capacity

- _ Immunity to crosstalk
- _ Immunity to static interference
- _ Environmental immunity
- _ Safety
- _ Security

19. Define a fiber optic system.

An optical communications system is an electronic communication system that uses light as the carrier of information. Optical fiber communication systems use glass or plastic fibers to contain light waves and guide them in a manner similar to the way electromagnetic waves are guided through a waveguide.

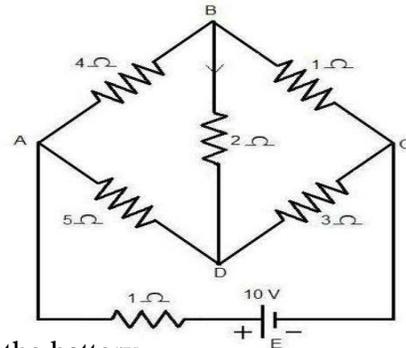
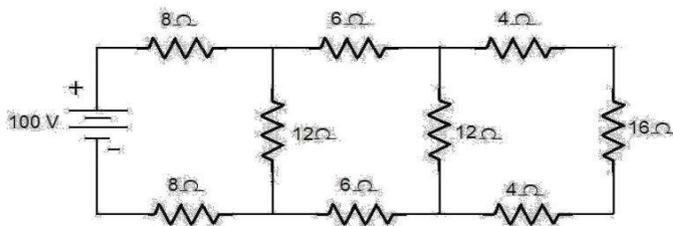
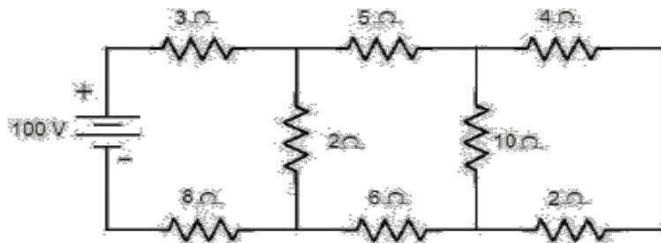
20. Distinguish between low level and high level modulation.

In low level modulation, modulation takes place prior to the output element of the final stage of the transmitter. It requires less power to achieve a high percentage of modulation. In high level modulators, the modulation takes place in the final element of the final stage where the carrier signal is at its maximum amplitude and thus , requires a much higher amplitude modulating signal to achieve a reasonable percent modulation

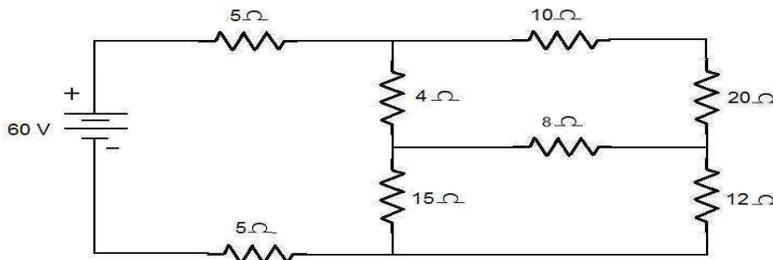
PART B

UNIT I-ELECTRICAL CIRCUITS AND MEASUREMENTS

1. State and explain Kirchoff's law.
2. Explain the working of a Dynamometer wattmeter with a neat sketch.
3. Explain any one type of MI instruments.
4. Explain the working principle of PMMC instruments.
5. Explain the construction and principle of operation of single phase energy meter.
6. (a) A series circuit has $R=10 \Omega$, $L=50\text{mH}$, and $C=100\mu\text{F}$ and is supplied with $200\text{V}, 50\text{Hz}$. Find (i) Impedance (ii) current (iii) power (iv) power factor (v) voltage drop across the each element.
(b) Derive the equation for equivalent resistance of number of resistors connected in Parallel and series.
7. A 400V is applied to three star connected identical impedances each consisting of a 40 resistance in series with 3 inductance reactance. Find (i) line current (ii) Total power supplied.
8. Find the current through each branch by network reduction technique.



9. (a) In the network shown below, find the current delivered by the battery



- (b) Discuss about voltage and current division principles.

UNIT II-ELECTRICAL MACHINES

1. Explain the construction and principle of operation of a DC generator with neat sketch.
2. Derive the equation for induced EMF of a DC machine.
3. Derive the torque equation of DC motor.
4. Describe the construction details of single phase transformer and also explain the principle of operation.
5. Derive the EMF equation of a transformer.
6. Explain the principle of operation of DC Motor.

7. Explain the construction and principle of operation of single phase induction motor.
8. A transformer with 40 turns on the high voltage winding is used to step down the voltage from 240V to 120V. Find the number of turns in the low voltage winding.
9. A 4 pole, wave wound generator having 40 slots and 10 conductors placed per slot. The flux per pole is 0.02 wb. Calculate the generated emf when the generator is drive at 1200 rpm.
10. A 25kw, 250V, dc shunt generator has armature and field resistances of 0.06ohm and 100ohm respectively. Determine the total armature power developed when working (1) as a generator delivering 25 kw output and (2) as a motor taking 25kw.
11. Draw the construction of DC machine and describe the working of the same with necessary diagram.

UNIT III-SEMICONDUCTOR DEVICES AND APPLICATIONS

1. Explain the operation of NPN and PNP transistor.
2. Describe the working of PN junction diode in forward and reverse bias conditions with neat diagrams. Also explain its V-I characteristics.
3. What is a Zener diode? Explain the operation of Zener diode and draw its Characteristics and show how it is used as a voltage regulator
4. Explain the operation of half wave rectifier and bridge rectifier with neat sketch and derive the necessary expression.
5. Explain the operation of centre tapped full wave rectifier with neat diagram.
6. Explain with neat diagram how the input and output characteristics of a CE configuration can be obtained.
7. Explain the input, output characteristics of CB, CC and CE configuration
8. Explain the working of the CB, CE configuration of a BJT.
9. Explain in detail about small signal CE amplifier.
10. Explain the mechanism of avalanche breakdown and zener breakdown.

UNIT IV-DIGITAL ELECTRONICS

1. Draw and explain the operation of AND, OR, NOT, NAND and NOR gates with suitable truth table.
2. Explain half adder and full adder.
3. Briefly explain the working of JK flip flop and the operation of RS flip-flop with logic diagram and truth table.
4. With necessary diagrams explain the functioning of the following:
 - i) Decade counter ii) D/A converter
5. What is a counter? Discuss briefly about Mod-5 counter
6. With necessary diagrams explain the functioning of any one type of A/D converter and D/A converter.
7. Draw the logic diagram of a 4 bit shift register and explain its working.
8. Describe the operation of a 4-bit binary, ripple counter.
9. Discuss about A/D conversion.
10. Show that NAND and NOR gates are universal building blocks.

UNIT V-FUNDAMENTALS OF COMMUNICATION ENGINEERING

1. What is the need for modulation? With neat diagram, explain the principle of operation of Amplitude Modulation.
2. Write detailed notes on microwave communication.
3. Explain a) satellite communication system. b) Balanced modulator
4. Write short notes on modulation and demodulation

5. Draw the block diagram of radio broadcasting and reception system and explain the function of each block.
6. With a neat block diagram, explain the principle of operation of FAX
7. Explain the block diagram of optical fiber communication systems.
8. Explain the principle of Frequency modulation. What are the advantages and disadvantages of frequency modulation over amplitude modulation?
9. Draw and explain the functional block diagram of Monochrome TV transmitter and receiver.
10. Explain about satellite communication

ENGINEERING MECHANICS

**V.S.B.ENGINEERING COLLEGE
DEPARTMENT OF CIVIL ENGINEERING
ENGINEERING MECHANICS
Two Marks Question Bank
UNIT-I**

1. Define Engineering Mechanics

Engineering Mechanics is defined as the branch of physical science which deals with the behaviour of a body at rest or motion under the action of forces.

8. What are the branches of Engineering Mechanics?

1. Rigid body Mechanics
2. Deformable body mechanics (also called strength of materials)
3. Fluid Mechanics

9. What are the branches of Rigid body Mechanics?

Statics
Dynamics

10. Define statics.

Statics is defined as the branch of rigid body mechanics, which deals with the behaviour of a body when it is at rest.

5. Define Dynamics

Dynamics is defined as the branch of rigid body mechanics which deals with the behaviour of a body when it is in motion.

6. Distinguish between particle and Rigid body

A body of negligible dimension is called a particle. A large number of particles which occupy fixed positions with respect to each other both before and after applying a load is called Rigid body

7. The Quantity which has only magnitude is called (Ans: Scalar)

8. Vector Quantity has both (Ans: Magnitude and direction)

9. Define Force.

Force is defined as an agent which changes or tends to change the state of rest or of uniform motion of a body. It represents the push or pull exerted by one body on another. It is a vector quantity.

10. What are the characteristics of a force?

1. Magnitude
2. Line of action
3. Direction & angle of inclination
- 4.

11. State Newton's laws of motion

Newton's first law: Everybody preserves in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed there on.

Newton's second law : The acceleration of a particle will be proportional to the force and will be in the direction of the force (ie. $F = ma$)

Newton's third law: To every action there is an equal and opposite reaction.

12. State the Principle of transmissibility.

It state that “any force at a point on a rigid body can be transmitted to act at any other point along its line of action without changing its effect on the rigid body”

13. What is collinear force system?

Force acts on a common line of action.

14. What is like parallel forces?

The parallel force which acts in the same direction are called like parallel forces.

15. What is unlike parallel forces?

The parallel force which acts in the opposite direction are called unlike parallel forces.

16. Two vectors are equal if

Ans: Their magnitudes, direction and the sense are the same and lie anywhere in space)

19. What is coplanar force system?

In coplanar force system, lines of action of all forces lie on a single plane.

20. What is Non-coplanar (or spatial) force system?

In Non-coplanar (or spatial) force system, lines of action of all forces lie on different planes

21. What is collinear force system?

In collinear force system, all the forces lie on a single line.

22. What is concurrent force system?

In concurrent force system, lines action of all forces intersects at a point.

23. What is parallel force system?

In parallel force system, lines of action of all forces are parallel to each other.

24. State Newton's law of Gravitation?

It states that two particles of mass m_1 and m_2 mutually attracted with equal and opposite forces.

25. State the difference between internal and external forces.

External forces: The forces which represent the action of other bodies on the rigid body considered and which are responsible for the external behaviour of the rigid body are called as 'External forces'.

Internal forces:

The forces which hold together the forming the rigid body or holding the component parts together are called as internal forces.

26. Define resultant force?

Resultant force is a single equivalent force which can replace the given force system for an equivalence of effect

27. State parallelogram law of forces?

It states that "If two forces acting simultaneously on a particle be represented in magnitude and direction by the two adjacent sides of a parallelogram their resultant may be represented magnitude and direction by the diagonal of the parallelogram which passes through their point of intersection.

28. State triangle law of forces?

It states that "If two forces acting simultaneously on a particle represented in magnitude and direction by the two sides triangle, taken in order, their resultant may be represented magnitude and direction by the third side of the triangle, taken opposite order".

29. State polygon law of forces?

It states that "If a number of forces acting simultaneously on a particle be represented in magnitude and direction, by the sides of a polygon taken in order, then the resultant of all these forces may be represented in magnitude and direction, by the closing side of the polygon, taken in opposite order".

30. State the principle of resolution?

The algebraic sum of the resolved parts of a number of forces in a given direction is equal to the resolved part of their resultant in the same direction of their resultant and in the same direction.

31. What is the significance of parallelogram law in statics of particles?

Parallelogram law is used to find the resultant of two concurrent coplanar forces It can be applied by both analytically and graphically.

32. Define equilibrium?

A body is said to be in a state of equilibrium, if the body is either at rest or is moving at a constant velocity.

33. State Lami's theorem?

It states that, "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two".

$$P/\sin\alpha = Q/\sin\beta = R/\sin\gamma$$

34. What is two force equilibrium principles?

If a body is in equilibrium acted upon by two forces they must be of collinear forces of equal magnitude and opposite sense.

21. What is three force equilibrium principles?

If a body is in equilibrium acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.

36. What is four force equilibrium principles?

If a body is in equilibrium, acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two.

37. What are the three equations of equilibrium?

1. $\sum H = 0$

The algebraic sum of the horizontal forces must be zero.
ie., sum of the left hand side forces must be equal to sum of the right hand side forces.

2. $\sum V = 0$

The algebraic sum of the vertical forces must be zero.
ie. Sum of the upward forces must be equal to sum of the downward forces

3. $\sum M = 0$

The algebraic sum of the moments about a point must be zero
ie., sum of the clockwise moments about a point must be equal to sum of the anticlockwise moments about the same Point.

38. What is stable equilibrium?

A body is said to be in stable equilibrium, if it returns back to its original position after it is slightly displaced from its position of rest.

39. What is unstable equilibrium?

A body is said to be in unstable equilibrium, if it does not return back to its original position and heels farther away after slightly displaced from its position of rest.

40. What is neutral equilibrium?

A body is said to be in neutral equilibrium, if it occupies a new position (also remains at rest) after slightly displaced from its position of rest..

41. What is Free body diagram?

It is a sketch of the particle which represents it as being isolated from its surroundings. It represents all the forces acting on it.

UNIT - II

Equilibrium of Rigid bodies

1. The position vector and force are $2i - 3j + 4k$ and $120i - 260j + 320k$ respectively. Find the moment of the Force about the origin. And also find the scalar quantity of the moment.

$$\begin{aligned}\vec{M}_O &= \vec{r} \times \vec{F} \\ &= 2i - 3j + 4k \times 120i - 260j + 320k \\ &= \begin{vmatrix} i & j & k \\ 2 & -3 & 4 \\ 120 & -260 & 320 \end{vmatrix} \\ &= 80i - 160j - 160k\end{aligned}$$

$$\begin{aligned}\text{Scalar quantity} &= \sqrt{M_x^2 + M_y^2 + M_z^2} \\ &= \sqrt{80^2 + (-160)^2 + (-160)^2} = 240 \text{ units}\end{aligned}$$

2. In the above problem, find the angles made by the moment along x, y and z axes

$$\begin{aligned}\phi_x &= \cos^{-1}(M_x/M) = \cos^{-1}(80/240) = 70.52^\circ \\ \phi_y &= \cos^{-1}(M_y/M) = \cos^{-1}(-160/240) = 131.8^\circ \\ \phi_z &= \cos^{-1}(M_z/M) = \cos^{-1}(-160/240) = 131.8^\circ\end{aligned}$$

3. Find the unit vector along the force $\vec{F} = 2i + 3j + 5k$

$$\begin{aligned}\text{Unit vector } \lambda &= 2i + 3j + 5k / \sqrt{2^2 + 3^2 + 5^2} \\ &= 0.324i + 0.489j + 0.811k\end{aligned}$$

4. Define the term couple?

A couple is that two forces are of equal magnitude opposite sensed parallel forces, which lie in the same plane.

5. What are the characteristics of a couple?

1. The algebraic sum of the forces is zero.
2. The algebraic sum of the moments of the forces about any point is the same and equal to the moment of the couple itself.

6. State Varignon's theorem?

Varignon's theorem: if a number of coplanar forces are acting simultaneously on a body, the algebraic sum of the moments of all the forces about any point is equal to the moment of the resultant force about the same point.

7. Define moment of a force?

The moment of a force about a point is defined as the turning effect of the force about that point.

$$\text{Moment} = \text{Force} \times \text{Perpendicular distance}$$

8. For what condition the moment of a force will be zero?

A force produces zero moment about an axis or reference point which intersects the line of action of the force.

9. What is the difference between a moment and a couple?

The couple is a pure turning effect which may be moved anywhere in its own plane, or into a parallel plane without change of its effect on the body, but the moment of a force must include a description of the reference axis about which the moment is taken.

10. What is the difference between a fixed vector and a free vector?

A force which is applied at a particular location on a body is a fixed vector.

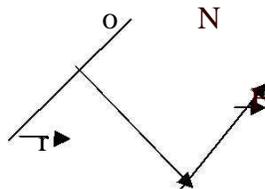
Example: A moment.

A force which can be moved anywhere in its own plane or in a parallel plane without change in its effect on the body is called free vector.

Example: A couple.

11. Explain the concept of moment of a force about an axis.

Moment of a force about an axis is a measure of the tendency of the force to rotate a body about that axis. If M_o is the moment of the force F about O , then moment of the force about the axis ON is,



$$M_{ON} = \lambda_{ON} \cdot M_o$$
$$M_o = r \times F$$

12. State the requirements for equilibrium of a body acted upon by a parallel force system?

1. The algebraic sum of the forces is zero. i.e., $\sum F=0$.
2. The algebraic sum of the moments about any point is zero. i.e., $\sum M=0$.

13. Draw the free body diagram of a ball (sphere) of weight W , resting on a frictionless plane surface shown below.



14. What are the necessary and sufficient conditions for the equilibrium of a rigid body in three dimensions?

$$\sum F_x = 0 \quad \sum M_x = 0$$
$$\sum F_y = 0 \quad \sum M_y = 0$$
$$\sum F_z = 0 \quad \sum M_z = 0$$

15. What are the common types of supports used in two dimensions?

- Roller support
- Hinged support
- Fixed support

16. What are the common types of supports used in three dimensions?

1. Ball support
2. Ball and Socket support
3. Fixed (or Welded) support

17. Define equilibrant?

The force which brings the system of forces into equilibrium is called equilibrant. It is equal to the resultant force in magnitude collinear but opposite in nature.

18. What are the common types of loads?

1. Point load (or concentrated load)
2. Uniformly distributed load
3. Uniformly varying load

19. What is statically determinate structure?

A structure which can be completely analysed by static conditions of equilibrium ($\sum H = 0$; $\sum V = 0$ and $\sum M = 0$) alone is statically determinate structure.

20. Write the equations of equilibrium of a rigid body in two dimensions.

$$\begin{aligned}\sum F_X &= 0 \\ \sum F_Y &= 0 \\ \sum M_Z &= 0\end{aligned}$$

UNIT -III**Properties of Surfaces and Solids****1. Define Centre of Gravity.**

Centre of Gravity is an imaginary point at which the entire weight of the body is assumed to act.

2. Define Centre of mass.

Centre of mass is the point where the entire mass of a body is assumed to be concentrated.

3. Define Centroid.

Centre of gravity of a plane figure is referred as centroid. Centroid is the point at which the entire area of the figure is assumed to be concentrated

4. Differentiate centroid and Centre of gravity

Centroid is the geometric property of geometrical figures line, area and volume. Centre of gravity is the physical property of a body like wire, rod, disc and solids

5. When centroid and centre of mass coincide?

Centroid and centre of mass coincide when the density of the material is uniform throughout the body.

6. State the methods of determining the centre of gravity?

1. By Geometrical considerations
2. Graphical method
3. Integration method
4. Method of moments

7. Write the expressions to find centroid of a composite plane figure?

$$\begin{aligned}\bar{X} &= \text{sum of first moment of the area about y axis} / \text{Total area} \\ &= a_1 x_1 + a_2 x_2 / a_1 + a_2\end{aligned}$$

$$\begin{aligned}\bar{y} &= \text{sum of first moment of the area about x axis} / \text{Total area} \\ &= a_1 y_1 + a_2 y_2 / a_1 + a_2\end{aligned}$$

8. State Pappus and Guldinus theorems.

Theorem I: The area of the surface generated by revolving a plane curve about a non intersecting axis in the plane of the curve is equal to the product of length of the curve and the distance travelled by the centroid G of the curve during revolution.

Theorem II: The volume of the solid generated by revolving a plane area about a non intersecting axis in its plane is equal to the product of area and length of the path travelled by centroid G of the area during revolution.

9. What is Axis of revolution?

The fixed axis about which a plane curve (may be of an arc, straight line etc.,) or a plane area is rotated is known as axis of revolution

10. Define Axis of Symmetry?

The axis about which similar configuration exist with respect to shape, size and weight on either side is known as axis of symmetry. It may be horizontal, vertical or inclined

11. Define moment of inertia of a body.

Moment of inertia (I) about an axis is the algebraic sum of the products of the elements of mass and the square of the distance of the respective element of mass from the axis.

12. Define Radius of gyration.

Radius of gyration of any Lamina defined as the distance from the elemental parts of the lamina would about a given axis may be given axis at which all the have to be placed, so as not to alter the moment of inertia about the given axis.

$$\text{Radius of gyration } k = \sqrt{I/A}$$

Where I = Moment of inertia

A = Total area of the plane

13. State parallel axis theorem.

Parallel axis theorem states that “ if the moment of inertia of a plane area about an axis through its centroid be denoted by IG, the moment of inertia of the area about an axis AB, parallel to the first and at a distance ‘h’ from the centroid is given by ,

$$I_{AB} = I_G + Ah^2$$

14. State perpendicular axis theorem.

It states that “if I_{XX} and I_{YY} be the moment of inertia of a plane section about two perpendicular axis meeting at ‘O’ the moment of inertia I_{ZZ} about the axis Z-Z perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by the relation,

$$I_{ZZ} = I_{XX} + I_{YY}$$

15. Define polar moment of inertia?

The second moment of area about a pole 'O' is called the polar moment of inertia (Ip).

$$I_p = I_{xx} + I_{yy}$$

16. Define principal axes and principal moment of inertia.

The maximum and minimum moment of inertia of a plane lamina are known as Principal moment of inertia. The axes about which the moment of inertia is either maximum or minimum are known as principal axes.

17. Write the S.I. units of area moment of inertia and mass moment of inertia of a lamina.

S.I. units of area moment of inertia of lamina are m⁴ and mass moment of inertia of a lamina is kg – m².

18. Differentiate between area moment of inertia and mass moment of inertia.

Area moment of inertia about any axis is the product of area and square of the distance of its centroid from that axis. Mass moment of inertia about any axis is the product of mass and square of the distance of its centroid from that axis.

19. When will be the product of inertia of lamina becomes zero?

The product of inertia of lamina becomes zero when either one of the two axes or both the axes are axis of symmetry.

20. Define first moment of an area about an axis.

The first moment of an area about an axis is the product of area and the

perpendicular distance of its centroid from the axis.

UNIT-IV Dynamics of Particles

1. Define 'speed

The rate of change of displacement of a body irrespective of its direction is called speed. It's a scalar quantity

2. Define velocity

The rate of change of displacement of a body with respect to its surroundings in a particular direction is called the velocity. It is a vector Quantity.

3. Define acceleration

The rate of change of velocity of a body is called acceleration.

4. Define uniform acceleration

If a body moves in such a way that its velocity changes equal in magnitude in equal intervals of time, the body is said to be moving with a uniform acceleration.

5. Define variable acceleration.

If a body moves in such a way that its velocity changes unequal in magnitude in equal intervals of time, the body is said to be moving with a variable acceleration.

6. Write the equations of plane motion?

1. $v = u + at$
2. $s = ut + \frac{1}{2}at^2$
3. $V^2 = u^2 + 2gh$

Where

v =Final velocity; u =Initial velocity; a =acceleration

t =time taken for displacement; S =distance travelled.

7. Write the equations of motion of a body under the force of gravity?

1. $v = u + gt$
2. $h = ut + \frac{1}{2}gt^2$
3. $V^2 = u^2 + 2gh$

8. Write the equations of motion of a body against the force gravity?

1. $v = u - gt$
2. $h = ut - \frac{1}{2}gt^2$
3. $v^2 = u^2 - 2gh$

9. Define projectile

A particle, moving under the combined effect of vertical and horizontal forces is called a projectile

10. Define trajectory

The path traced by a projectile in the space is known as trajectory.

11. Define Angle of projection

The angle with the horizontal, at which a projectile is projected is known as the angle of projection.

12. Define Range of projectile

The distance between the point of projection and the point where the projectile strikes the ground is known as the Range of projectile.

13. Define velocity of projection?

The velocity with which a projectile is projected is called velocity of projection.

14. Define Time of Flight?

The total time taken by a projectile to reach maximum height and return back to the ground is known as time of flight.

15. Define energy?

It is the capacity to do work

16. Define potential energy?

It is the energy possessed by a body, for doing work, by virtue of its position

17. Define kinetic energy?

It is the energy possessed by a body, for doing work, by virtue of its motion.

18. State the law of conservation of energy?

It states that, "The energy can neither be created nor destroyed, though it can be transformed from one form into any of the form in which the energy can exist.

19. Define power.

The rate of doing work is called power

20. What is rectilinear motion?

The motion of a particle is said to be rectilinear, if it moves along a straight line.

21. What is curvilinear motion?

The motion of a particle is said to be curvilinear, if it moves along a curved path.

22. State the principle of conservation of linear momentum.

It states that, if the resultant force acting on a particle is zero, then the linear momentum of the particle remains constant ie, Final momentum = Initial momentum.

23. States Law of conservation of angular momentum.

The sum of the moments about 'O' of the forces acting on the particle is equal to twice the rate of change of angular momentum of the particle about 'O')

24. State the principle of work and energy.

It states that "when a particle moves from position, S1 to S2 under the action of a force F, the change in kinetic energy of the particle is equal to the force F".

25. What is conservative force?

A force F is said to be conservative, when the force components are derivable from a potential and the work done by the force F between any two points is independent of the path followed

26. States Theorem of conservation of energy.

When a particle is acted upon by conservative forces, the sum of the particle's kinetic and potential energy remains constant during the motion

27. Define Impulse of a force.

When a large force acts over a short period of time, that force is called an impulsive force.

28. What is co-efficient of restitution?

The ratio of the magnitude of the impulses during the restitution period and deformation period is known co-Efficient of restitution.

29. Define Time of restitution?

It is defined as the time taken by two bodies to regain the original shape, after impact.

30. What is hodograph?



Let a particle has a velocity V at time t and a velocity $V' (= V + I \cdot V)$ at P and P' respectively as shown in fig (a). To study the time rate of change, the two velocity vectors are plotted such that their tails are located at the fixed point 'O' and their arrow heads touch points on the dashed curve as shown in fig (b) This curve is called as Hodograph.

31. Define average velocity.

It is the ratio of displacement and time interval.

Change in position/ Change in time

It can be either positive or negative.

32. Define Instantaneous velocity

It is the limit of average velocity as the increment of time approaches zero. It can be either positive or negative.

33. State D' Alembert's principle.

It states that "The system of forces acting on a body in motion is in dynamic equilibrium, with the inertia force of the body".

34. What is work-energy equation?

$$P \cdot S = W/2g(v^2 - u^2)$$

P- Force, S-distance travelled

W- Weight of the body, g- acceleration due to gravity v- Final velocity, u- initial velocity

35. What is Impulse - momentum equation?

$$F \cdot t = m(v - u)$$

F- Impulsive force; m- Mass of the body; v- Final velocity

u- Initial velocity

36. What is line of impact?

It is an imaginary line passing through the point of contact and normal to the plane of contact.

37. What is direct impact?

In direct impact, the velocities of the two colliding bodies, before collision are collinear with the line of impact.

38. What is oblique Impact?

In oblique Impact, the velocities of the two colliding bodies, before collision are not collinear with the line of impact

39. What is central impact?

In central impact, the line of impact passes through the mass centres of the bodies

40. What is Non-central Impact?

In Non - central impact, the line of impact does not passes -through the mass centres of the bodies.

41. What is direct central impact?

In direct central impact, the velocities of the colliding bodies are directed along the line of impact and the line of impact passes through the mass centres of the colliding bodies

42. What is oblique central impact?

In Oblique central impact, the velocities of the colliding bodies are not directed along the line of impact but the line of impact passes through the mass centres of the colliding bodies.

43. State Newton's law of collision.

It states that "for two colliding bodies, their relative velocity of separation bears a constant ratio to their relative velocity of approach".

UNIT – V

Friction

1. Define friction

Friction may be defined as a force of resistance acting on a body which prevents or retards slipping of the body relative to a second body or surface with which It is in contact

2. Define static friction

Static friction between two bodies is the tangential force which opposes the sliding of one body relative to the other.

3. Define Dynamic friction or Kinetic Friction.

Dynamic friction is the tangential force between two bodies after motion begins

4. Define Angle of Friction.

Angle of friction is the angle between the line of action of the total reaction of one body on another and the normal to the common tangent between the' bodies when motion is impending.

5. Define Limiting Friction

Limiting friction 'F' is the maximum value of static friction that occurs when motion is impending.

6. Define Co-efficient of static friction.

Coefficient of static friction is the ratio of the Static friction to the normal reaction

7. Define coefficient of Dynamic friction.

Coefficient of Dynamic friction is the ratio of the Dynamic friction to the normal reaction

8. Define Angle of repose

Angle of repose (α) is the angle to which an inclined plane may be raised before an object resting on it will move under the action of the force of gravity

9. Define cone of friction.

It is defined as the right circular cone with vertex at the point of contact of the two bodies (or surface) axis in the direction of normal reaction (R) and semi vertical angle equal to angle of friction.

10. Define Solid Friction or Dry Friction.

If between two surfaces, no lubrication (oil or grease) used, the friction that exists between two surfaces is called solid friction.

11. What is the sliding friction?

It is the friction, experienced by a body when it slides over another body.

12. What is Rolling Friction?

It is the friction, experienced by a body when it rolls over the other.

13. State the Laws of static friction?

- a) The force of friction always acts in a direction opposite to that in which the body tends to move.
- b) The Magnitude of the force of friction is equal to the force, which tends to move the body.
- c) Limiting friction bears a constant ratio to the normal reaction between the two surfaces
- d) The force of friction is independent of the area of contact between the two surfaces
- e) The force of friction depends upon the roughness of the surfaces.

14. State the laws of Dynamic friction?

- a) The force of friction always acts in a direction, opposite to that in which the body is moving.
- b) The magnitude of the kinetic friction bears a constant ratio to the normal reaction between the two surfaces.
- c) For moderate speeds, the force of friction remains constant and it decreases with the increase of speed.

15. What is impending motion?

The motion is said to be impending if the applied forces are such that the body is just about to slide

16. State the laws of solid friction.

- a) The force of friction always acts in a direction opposite to that in which the body tends to move.
- b) The Magnitude of the force of friction is equal to the force, which tends to move the body.
- c) Limiting friction bears a constant ratio to the normal reaction between the two Surfaces.
- d) The force of friction is independent of the area of contact between the two surfaces
- e) The force of friction depends upon the roughness of the surfaces.

17. State the Laws of Dynamic friction.

- a) The force of friction always acts in a direction, opposite to that in which the body is moving.
- b) The magnitude of the kinetic friction bears a constant ratio to the normal reaction between the two surfaces.
- c) For moderate speeds, the force of friction remains constant and it decreases with the increase of speed.

18. When a screw is said to be self locking?

If the friction angle is larger than the lead angle of screw, the load will remain in plane even after the removal of effort. This Condition is said to be self locking.

19. What is meant by the term ‘translation’?

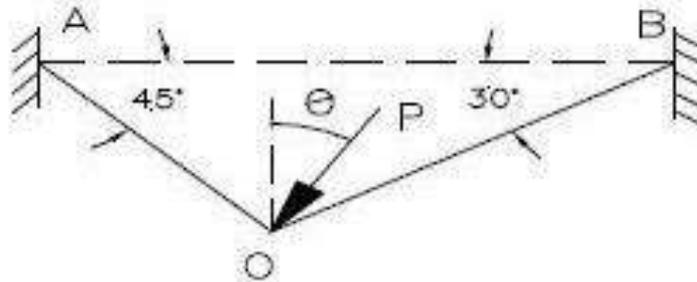
The type of motion of a body is said to be translation if the linear displacement of every point in the rigid body is the same.

20. What is general plane motion?

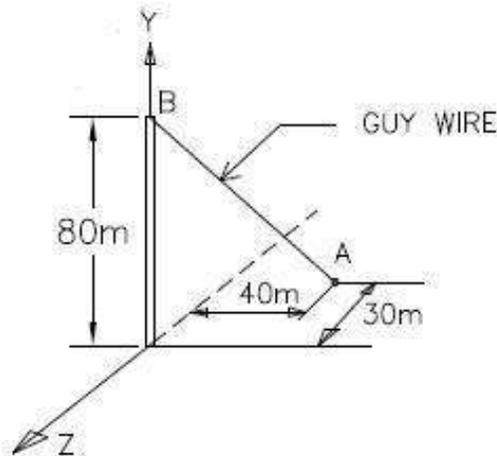
A rigid body is said to be in General plane motion, if it undergoes a combination of translation through some distance and also rotate through a certain angle.

QUESTION BANK
UNIT I -PART-B

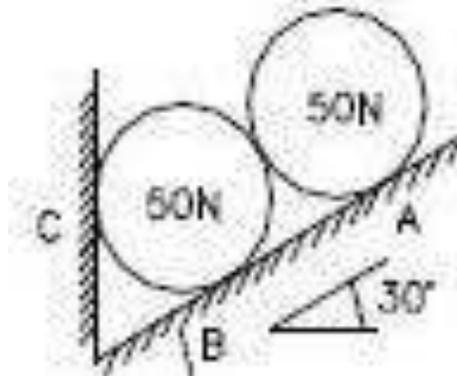
1. A force P is applied at 'O' to the string AOB as shown in fig. If the tension in each part of string is 50 N, Find the direction and magnitude of force P for equilibrium conditions. BTL1



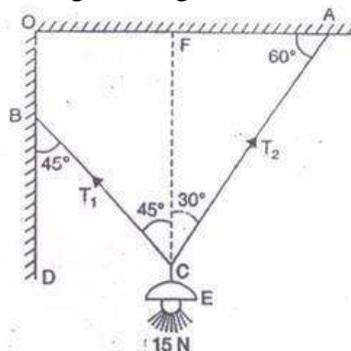
2. A tower guy wire shown below is anchored by means of a bolt at A as shown. The tension in the wire is 2500kN. Find (a). The components F_x , F_y & F_z of the force acting on the bolt (b). The angles α_x , α_y , α_z defining the direction of the force



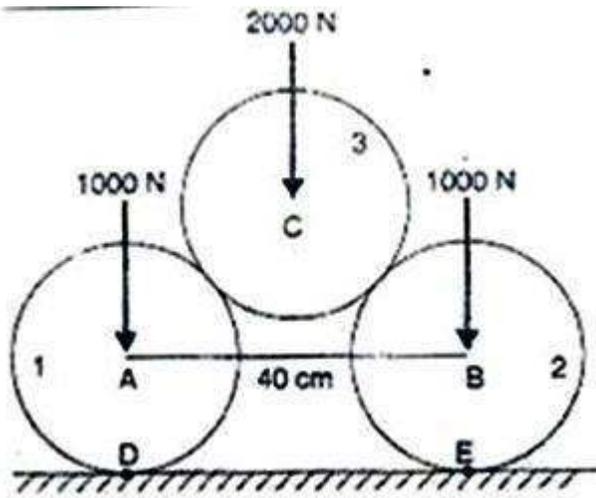
3. Two identical rollers each of weight 50N are supported by an inclined plane and a vertical wall as shown in fig. Find the reactions at the points of supports A, B, and C. BTL1



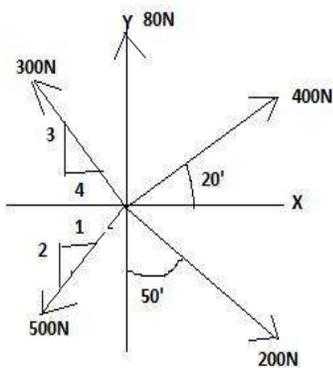
4. Predict the tension in each cable for the given Figure



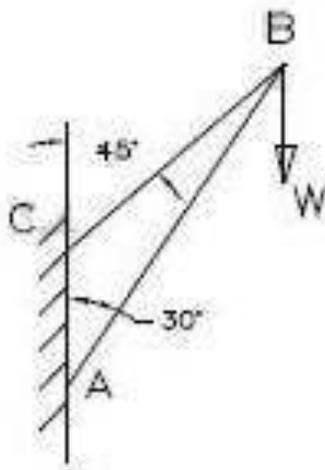
5. Two smooth circular cylinders each of weight 1000 N and radius 15 cm are connected at their centers by a string AB of length 40 cm and rest upon a horizontal plane, supporting above them a third cylinder of weight 2000 N and radius 15 cm as shown in Figure. Predict the force S in the string AB and reactions on the floor at the points of contact D and E.



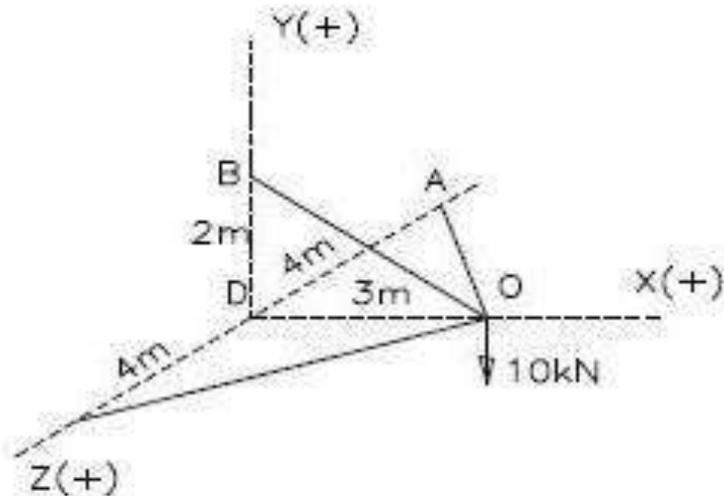
6. Determine the resultant of system of forces acting as shown in fig. BTL3



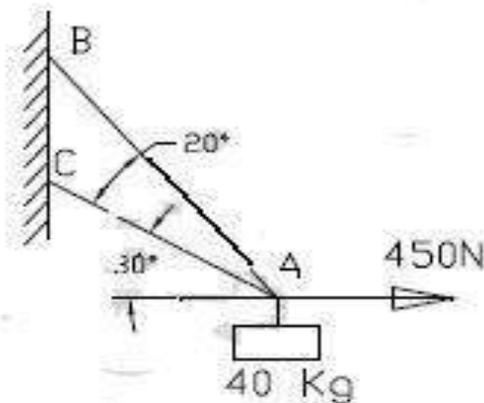
7. A crane shown in figure is required to lift a load of $W=10$ KN. Solve the forces in the members AB and CB



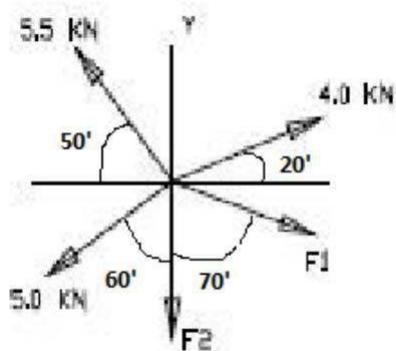
8. Members OA, OB and OC form a three member space truss. A weight of 10 kN is suspended at the joint 'O' as shown in fig. Analyze magnitude and nature of forces in each of the three members of the truss.



9. Design the tension in cables AB & AC to hold 40 Kg load shown in fig.

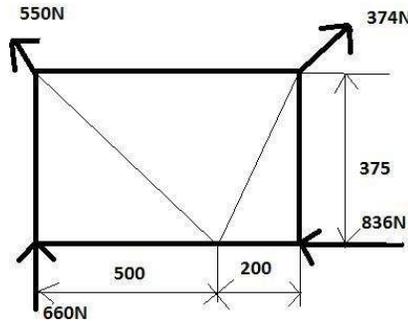


10. The forces shown in the figure below are in equilibrium. Determine the forces F1 and F2

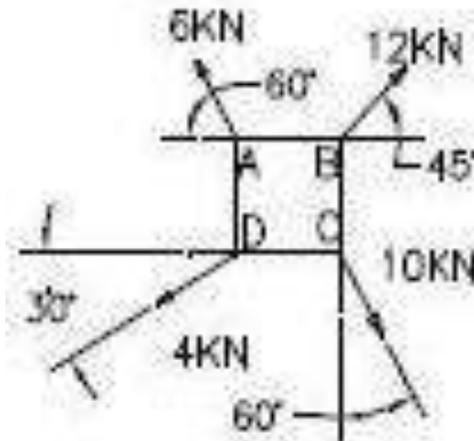


UNIT II - PART-B

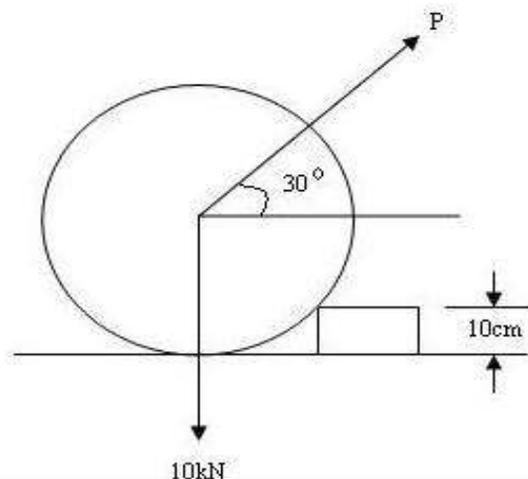
1. Four forces act on a 700mm X 375mm plate as shown in fig. a) Find the resultant of these forces b) Locate the two points where the line of action of the resultant intersects the edge of the plate.



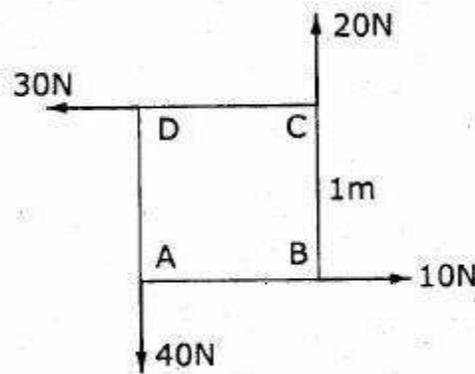
2. Four coplanar non concurrent non parallel forces act on a square plate of side 2m as shown in fig. Find the resultant forces.



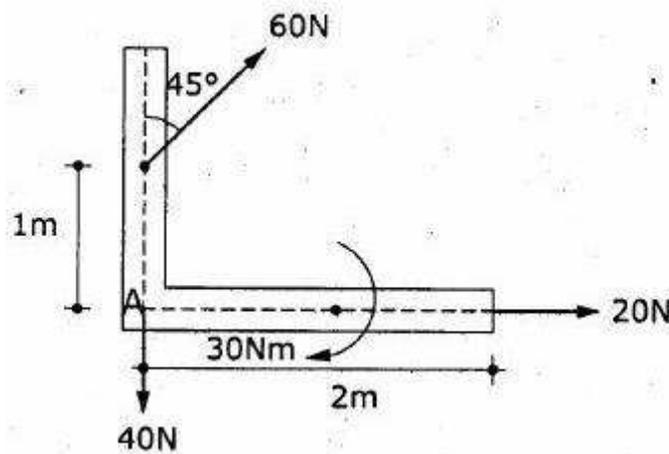
3. A cylindrical roller has a weight of 10kN and it is being pulled by a force which is inclined at 30° with the horizontal as shown in fig. While moving it comes across an obstacle of 10cm height. Predict the force required to cross this obstacle when the diameter of the roller is 70cm.



4. Four forces act on a square of side 1 m as shown in fig. Predict the force system into an equivalent force – couple system at A.

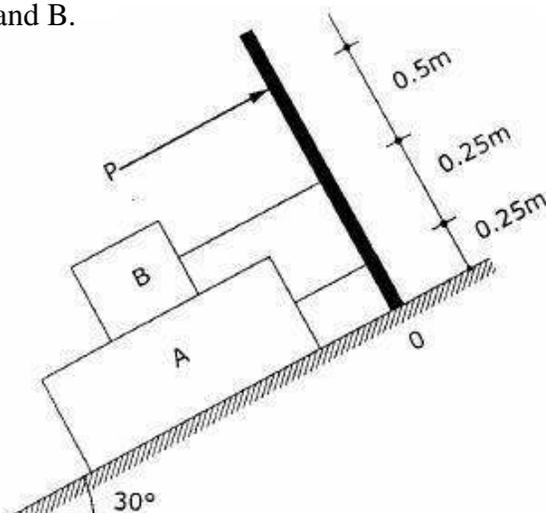


5. Illustrate the system of forces shown in fig to a force – couple system at A

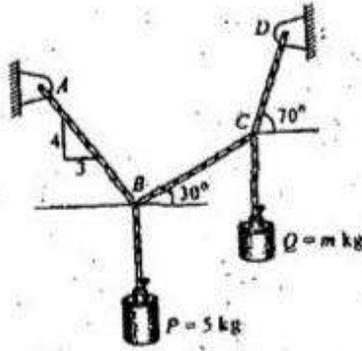


6. Blocks A and B of weight 200N and 100N respectively, rest on a 30° inclined plane and are attached to the post which is held perpendicular to the plane by force P, parallel to the plane, as shown in fig. Assume that all surfaces are smooth and that the cords are parallel to the plane. Determine the value of P. Also find the

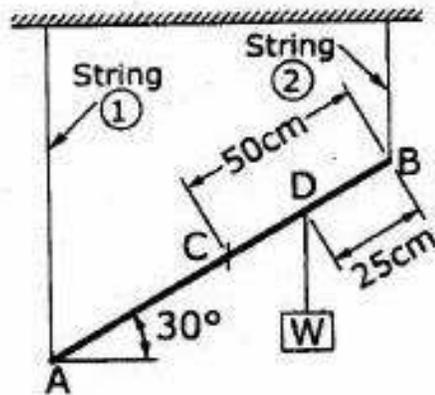
Normal reaction of Blocks A and B.



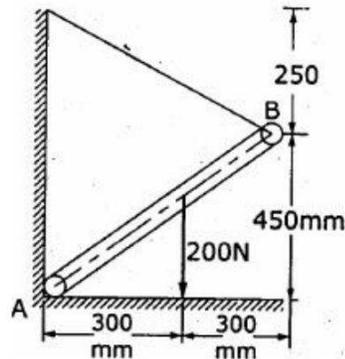
7. Block P = 50Kg and Block Q of mass m Kg is suspended through the chord is in the equilibrium position as shown in fig. Determine the Block of Mass Q.



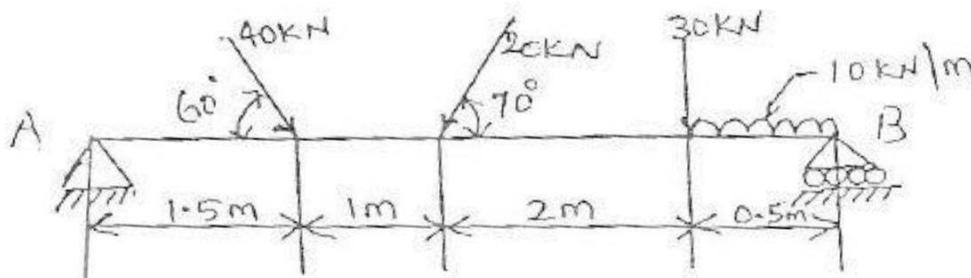
8. A Uniform meter rod AB, assumed rigid of mass 0.5 kg is suspended from its ends in an inclined position and a mass of 1 kg is suspended from a point D, as shown in fig. Determine the tension in each string. Where the suspended mass should be placed in order to get equal tension in the strings.



9. A rod AB of weight 200 N is supported by a cable BD and the corner of wall and floor surface as shown in fig. Show that the reaction at A and tension in the cord.

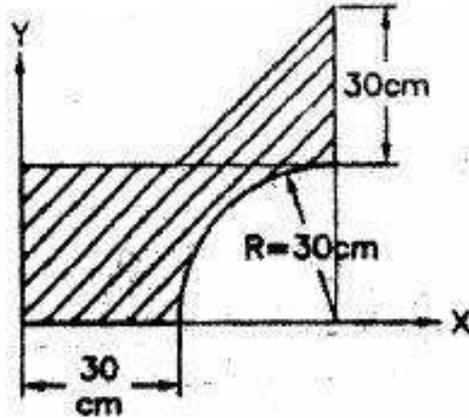


10. Solve reactions at points A & B



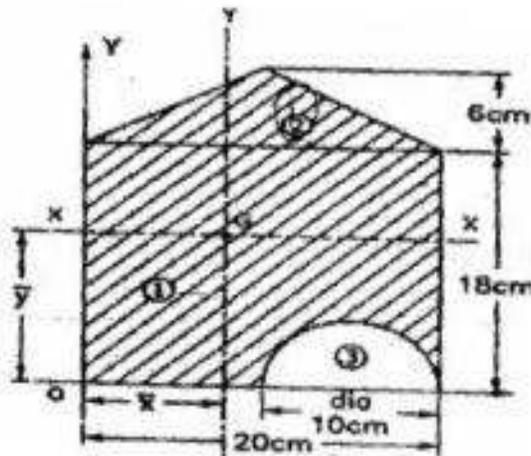
UNIT III - PART-B

1. Determine the co-ordinates of centroid of the shaded area shown in figure.

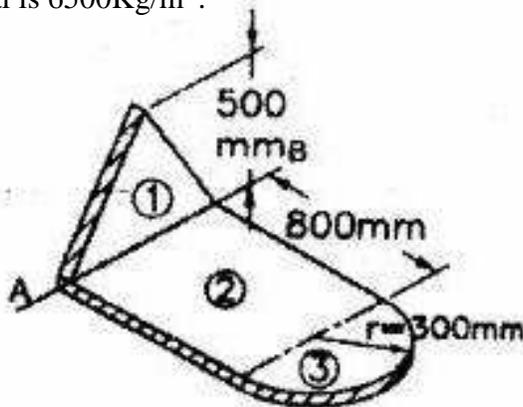


2. A Cylinder of height of 10 cm and radius of base 4 cm is placed under sphere of radius 4 cm such that they have a common vertical axis. If both of them are made of the same material, find the centre of gravity of the combined unit.

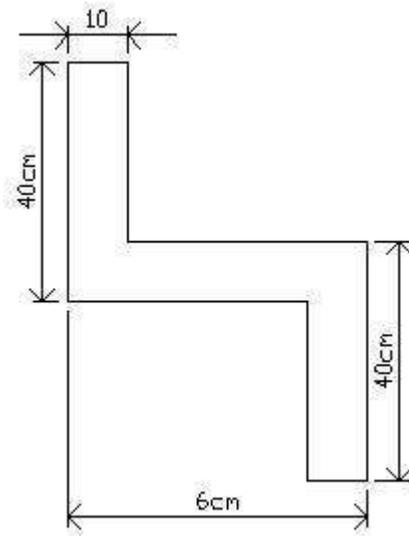
3. Find the moment of inertia of the section shown in the figure about its horizontal centroidal axis.



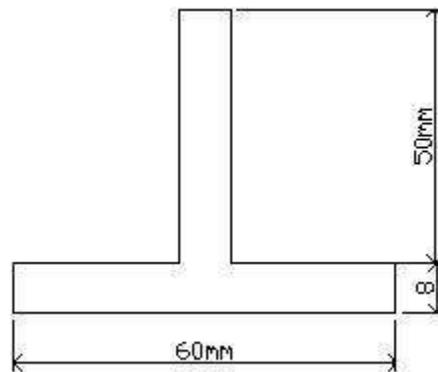
4. Find the mass moment of inertia of the plate shown in fig with respect to the axis AB. Thickness of the plate is 5mm and density of the material is 6500 Kg/m^3 .



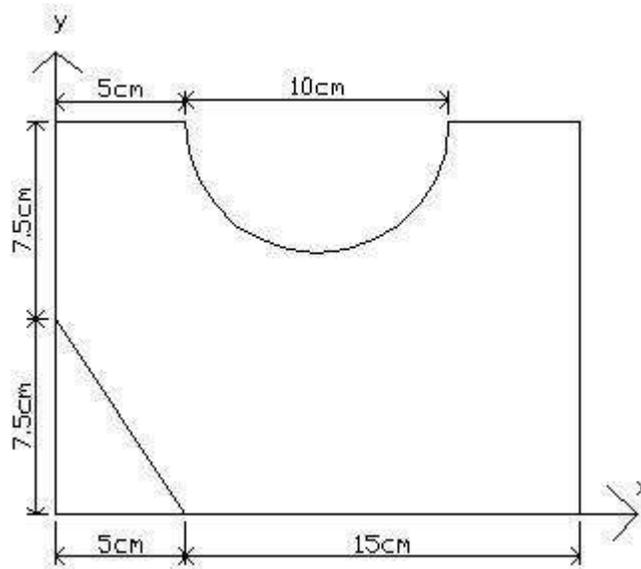
5. Discuss expression form mass moment of inertia of prism along three axes.
6. Design Moment of Inertia about the co-ordinate axes of plane area shown in fig. Also find Polar Moment of Inertia.



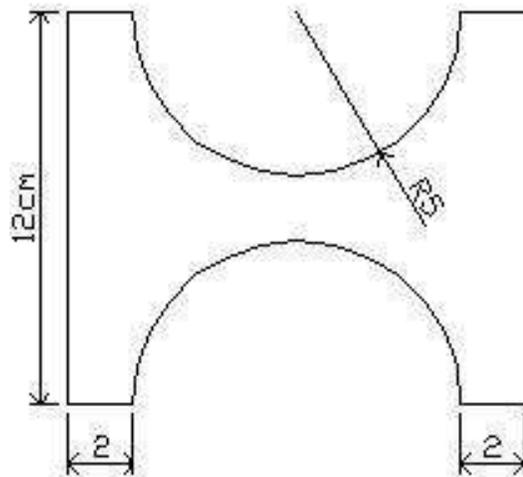
7. Determine the principal moments of inertia and find location of principal axes of surface shown in fig.



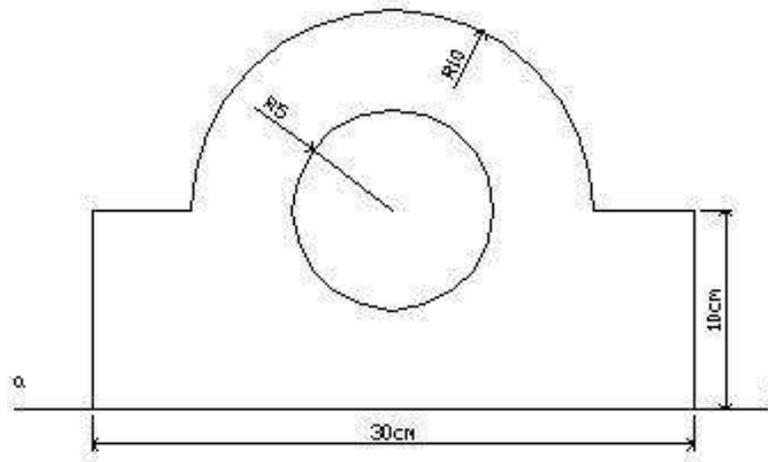
8. Illustrate Moment of Inertia and radius of gyration of surface about x axis shown in fig. Also find MOI about centroidal x axis.



9. Illustrate the polar moment of inertia and polar radius of gyration of plane area about centroidal axes shown in fig.



10. Explain second moment of area about the centroidal XX axis and a-a axis of the surface shown in fig.



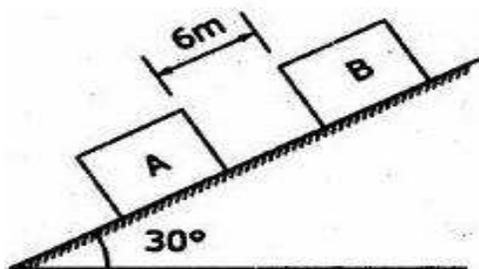
UNIT IV - PART-B

1. A train is traveling from A to D along the track shown in fig. Its initial velocity at A is zero. The train takes 5 min to cover the distance AB, 2250 m length and 2.5 minutes to cover, the distance BC, 3000 m in length, on reaching the station C, the brakes are applied and the train stops 2250 m beyond, at D (i) Find the retardation on CD, (ii) the time it takes the train to get from A to D, and (iii) its average speed for the whole distance.

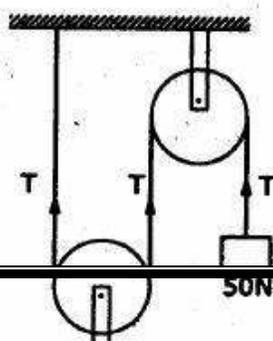
2. The position of the particle is given by the relation $S=1.5t^3-9t^2-22.5t+60$, where S is expressed in meters and t in seconds. Determine (i) the time at which the velocity will be zero (ii) the position and distance traveled by the particle at that time (iii) the acceleration of the particle at that time and (iv) the distance traveled by the particle from $t = 5s$ to $t = 7s$.

3. A particle is projected with a initial velocity of 12m/s at an angle M with the horizontal. After sometime, the position of the particle is observed by its x and y distances of 6m and 4m respectively from the point of projection. Find the angle of projection.

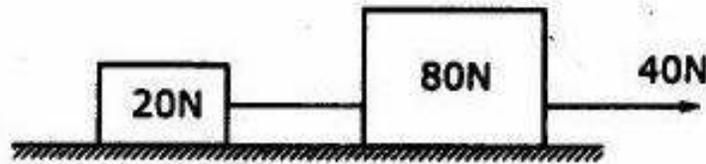
4. Two Blocks A and B of weight 100 N and 200 N respectively are initially at rest on a 30° inclined plane as shown in figure. The distance between the blocks is 6 m. The coefficient of friction between the block A and the plane is 0.25 and that between the block B and the plane is 0.15. If they are released at the same time, in what time the upper block (B) reaches the Block (A).



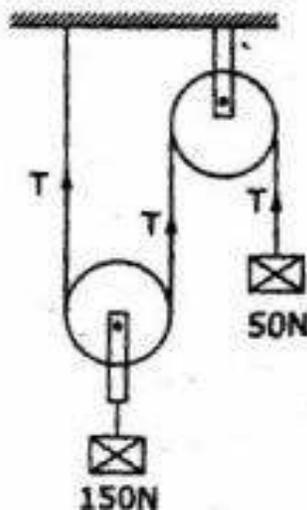
- Two blocks of weight 150 N and 50 N are connected by a string and passing over a frictionless pulley as shown in figure. Predict the acceleration of blocks A and B and the tension in the string.



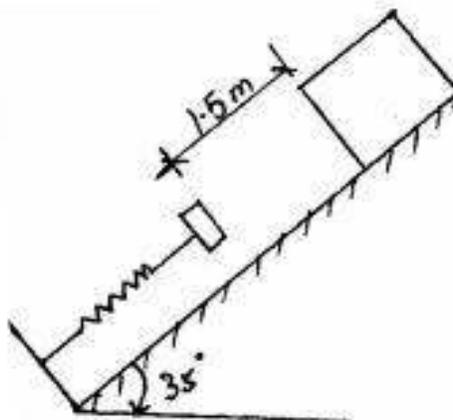
3. Two weights 80 N and 20 N are connected by a thread and move along a rough horizontal plane under the action of a force 40 N, applied to the first weight of 80 N as shown in figure. The coefficient of friction between the sliding surfaces of the weights and the plane is 0.3. Design the acceleration of the weights and the tension in the thread using work-energy equation.



4. Two blocks of weight 150N and 50N are connected by a string, passing over a frictionless pulley as shown in fig. Predict the velocity of 150N block after 4 seconds. Also calculate the tension in the string.



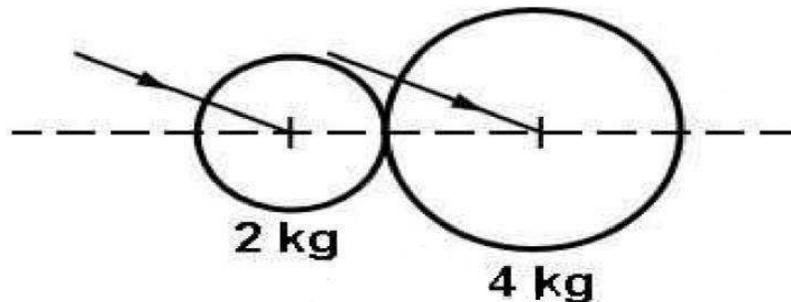
- 2 A block of mass 50 kg slides down a 35° incline and strikes a spring 1.5 m away from it as shown in Fig. The maximum compression of the spring is 300 mm when the block comes to rest. If the spring constant is 1 kN/m, Solve the coefficient of kinetic friction between the block and the plane.



9. Two bodies one of mass 30kg, moves with a velocity of 9m/s centrally. Solve the velocity of each body after impact, if the coefficient of restitution is 0.8

23. A ball of mass 2 kg, moving with a velocity of 3 m/s, impinges on a ball of mass 4 kg moving with a velocity of 1 m/s. The velocities of the two balls are parallel and inclined at 30° to the line of joining their centres at the instant of impact. If the coefficient of restitution is 0.5, Explain

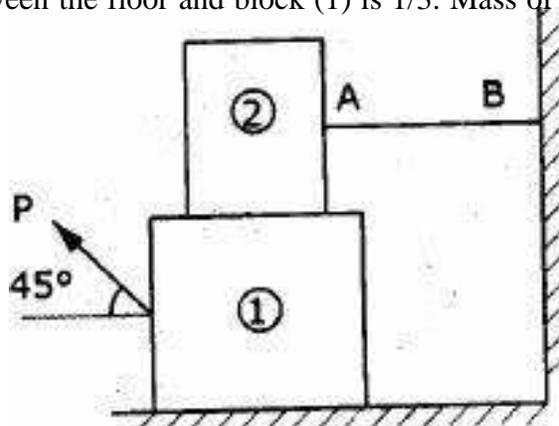
- I. Direction, in which the 4 kg ball will move after impact; (8)
- II. Velocity of the 4 kg ball after impact; (2)
- III. Direction, in which the 2 kg ball will move after impact; (4)
- IV. Velocity of the 2 kg ball after impact. (2)



UNIT V - PART-B

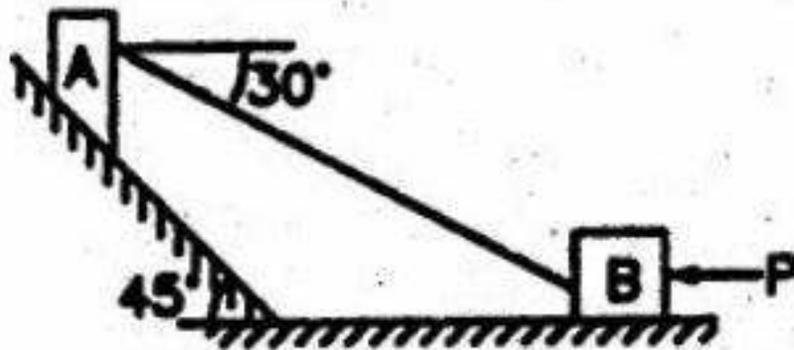
7. Block (2) rests on block (1) and is attached by a horizontal rope AB to the wall as shown in fig. What

force P is necessary to cause motion of block (1) to impend? The co-efficient of friction between the blocks is $\frac{1}{4}$ and between the floor and block (1) is $\frac{1}{3}$. Mass of blocks (1) and (2) are 14kg and 9 kg respectively.



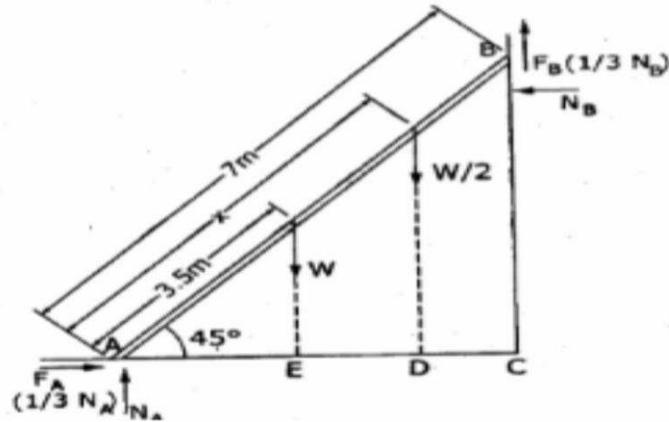
2. Block A weighing 1000 N rests on a rough inclined plane whose inclination to the horizontal is 45° . It is connected to another block B, weighing 3000 N rests on a rough horizontal plane by a weightless rigid bar inclined at an angle of 30° to the horizontal as shown in fig. Find the horizontal force

required to be applied to the block B just to move the block A in upward direction. Assume angle of friction as 15° at all surfaces where there is sliding.

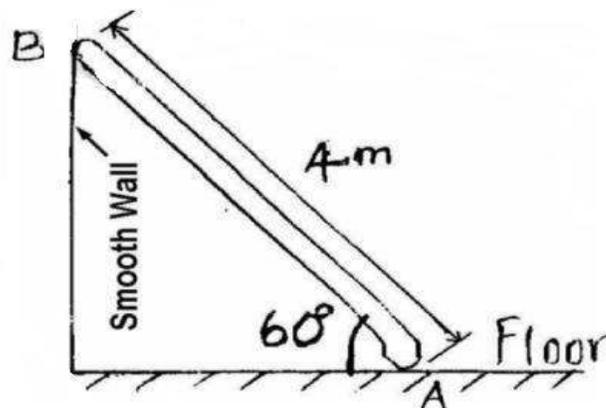


8. A 7m long ladder rests against a vertical wall, with which it makes an angle of 45° and on a floor. If a man whose weight is one half that of the ladder climbs it, at what distance along the ladder will he

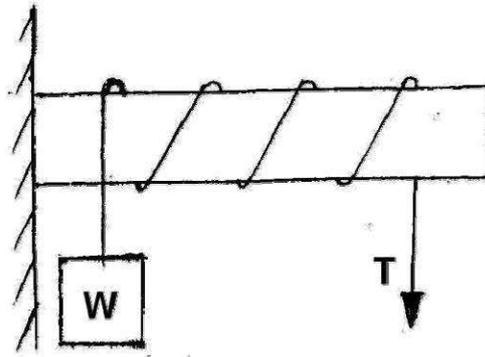
be, when the ladder is about to slip? Take coefficient of friction between the ladder and the wall is $\frac{1}{3}$ and that between the ladder and the floor is $\frac{1}{2}$.



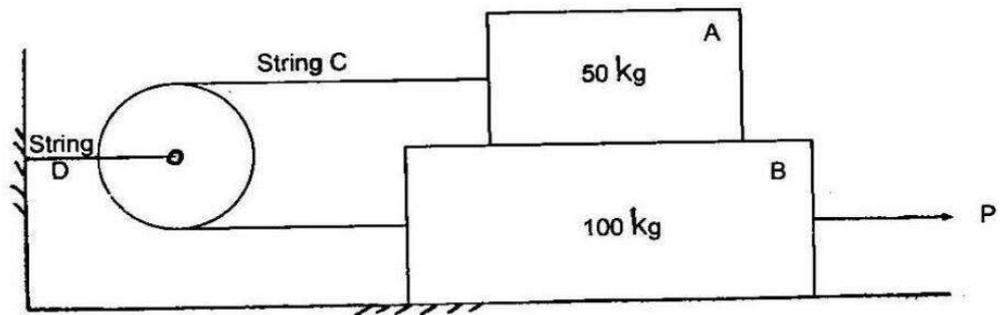
- Y In a screw jack, the pitch of the square threaded screw is 5.5 mm and means diameter is 70 mm. The force exerted in turning the screw is applied at the end of lever 210 mm long measured from the axis of the screw. If the co-efficient of friction of the screw jack is 0.07, Predict the force required at the end of the lever to (i) raise a weight of 30 KN (ii) lower the same weight.
- z An effort of 200 N is required just to move a certain body up an inclined plane of angle 15° , the force is acting parallel to the plane. If the angle of inclination of the plane is made 20° , the effort required being again parallel to the plane, is found to be 230 N. Predict the weight of the body and coefficient of friction.
6. Illustrate the force P inclined at an angle of 32° to the inclined plane making an angle of 25 degree with the horizontal plane to slide a block weighing 125 KN (i) up the inclined plane (ii) Down the inclined plane, when $P = 0.5$
7. A ladder of weight 1000 N and length 4 m rests as shown in figure. If a 750 N weight is applied at a distance of 3 m from the top of ladder, it is at the point of sliding. Determine the coefficient of friction between ladder and the floor.



8. A rope is wrapped 3 times around the rod as shown in the fig. Design the force required at the free end of the rope to stop the load $w=20\text{KN}$. Take $\mu=0.3$



9. Two blocks A and B of mass 50 kg and 100 kg respectively are connected by a string C which passes through a frictionless pulley connected with the fixed wall by another string D as shown in figure. Find the force P required to pull the lock B. Also find the tension in the string D. Take coefficient of friction at all contact surfaces as 0.3.



10. In the engine system shown in figure, the crank AB has a constant clockwise angular speed of 3000 r.p.m. For the crank position indicated, Analyze (i) the angular velocity of the connecting rod BP (ii) velocity of piston P

